MOBILITY MODELS
FOR
MOBILITY MANAGEMENT

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Zakopane, Poland

Vicente Casares-Giner

Universidad Politécnica de Valencia (Spain)

Mobility Model for Mobility Management

OUTLINE

- General aspects: scenarios, cells, ….
- Residence time and call holding times.
  - *Cell and handoff areas.*
- MM in location management.
  - *Tessellation of the plane.*
  - *Random walk.*
  - *Brownian motion.*
  - *Gauss-Markov.*
- Advanced synthetic models
- Macroscopic vision. Transportation theory.
  - *Fluid flow models.*
  - *Gravity models.*
  - *Rush hour versus busy hour.*
  - *Flow, Speed and Density.*

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MMMM

- *Mobility Models* are essential to perform analysis on wireless networks

- They have influence in *Mobility Management*:
  - Mobility tracking
    - *Location management.*
   - *Resource management (for handovers).*

Scenario for mobility models

- Different geographical areas
  - Hot spots
  - Urban
  - Highway/main roads
  - Open (rural)
- Tools
  - Markovian models
  - Transportation theory models
  - Flow traffic models.
- Analysis
  - Individual behavior $\leftarrow$ $\rightarrow$ Global (macroscopic) parameters
- Time dependency
  - Stationary
  - Non stationary

Type of mobility models

- Markovian
  - Characterization of individual movement behavior,
  - Random walk, random waypoint, Brownian motion,…
  - At microscopic level

- Fluid flow
  - Characterization of aggregated traffic, ..
  - At macroscopic level

- Gravity models
  - Characterization of flow of vehicles, person, ...
 - Very useful in transportation theory

- **But they can be related**:

Markovian models

Gravity models   **Fluid flow models**
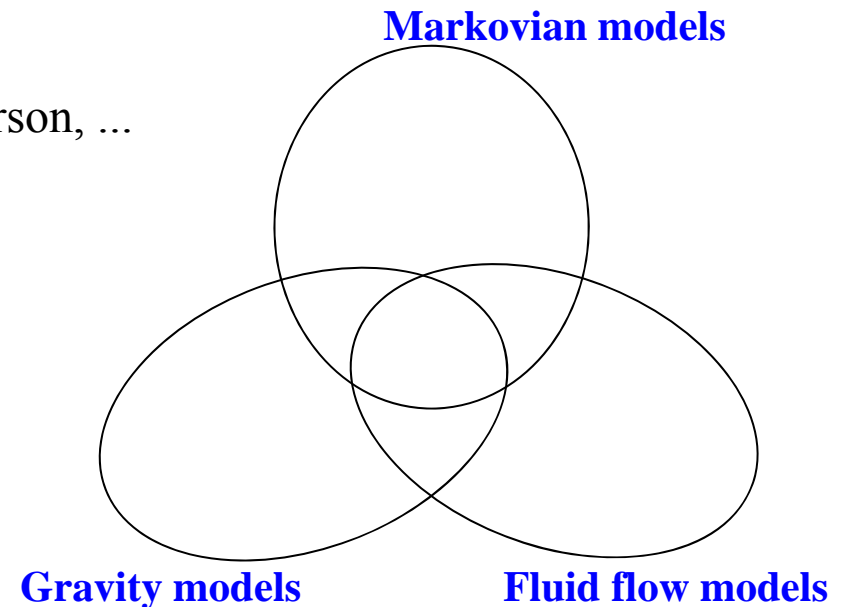
Type of cells

- **Macro-cells** (1 km – 40 km)
    - Rural areas, umbrellas in urban areas, …
    - Omni-directional 360º, sectors (120º, 60º)

- **Microcells** (100 m – 1 km)
    - Streets, main roads, avenues, ..
    - Cigar shape beam along the highway, …

- **Picocells** (10 m – 100 m)
    - Airports, railway stations, business areas, indoors, ...
    - Less regular in shape

Cell shape

- ## Irregular shape in practice

  - Features such as propagation, shadowing, etc.

- ## Regular shape when modeling

  - Triangle, square, hexagon, ..
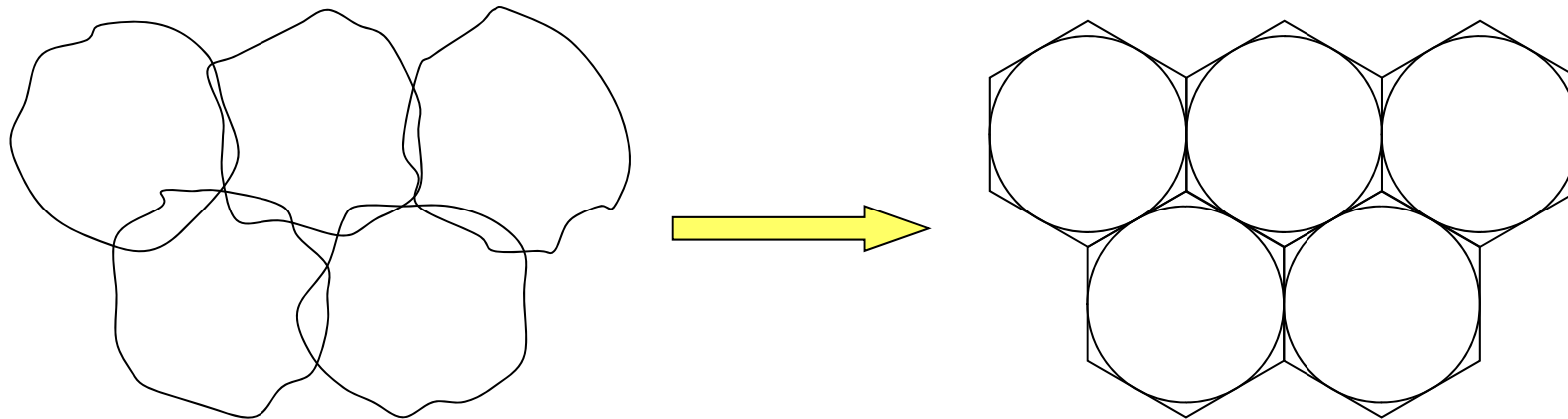
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Types of Mobility Models

- **Traces**
  - Traces are mobility patterns that are captured from the observation of realistic human trip movements.
  - They are quite complex to characterize and there results are rather difficult to compare.

- **Synthetic (Analytic ,…)**
  - Attempt to realistically represent the behaviors of MTs.
  - They are based on simple driven parameters, then easier to manage compared with trace models.
  - Examples: Random Walk (RW), Random Waypoint (RWP), Random Direction (RD), Gauss-Markov (GM), …..

Cell residence time

- ## Cell residence time
  - ### Depends on movement related parameter:
    - Speed
    - Cell geometry
    - Geographical environment
    - Trajectory of the mobile user
    - …..

T_R = Cell residence time
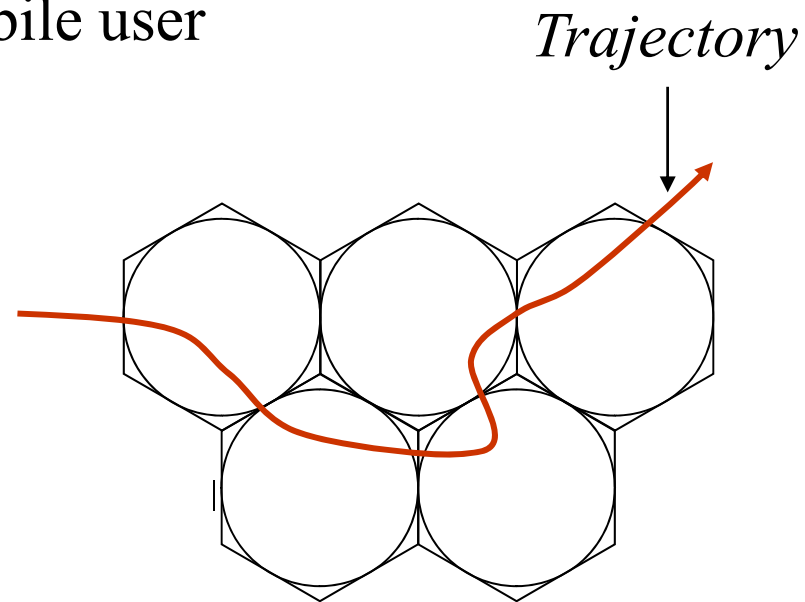
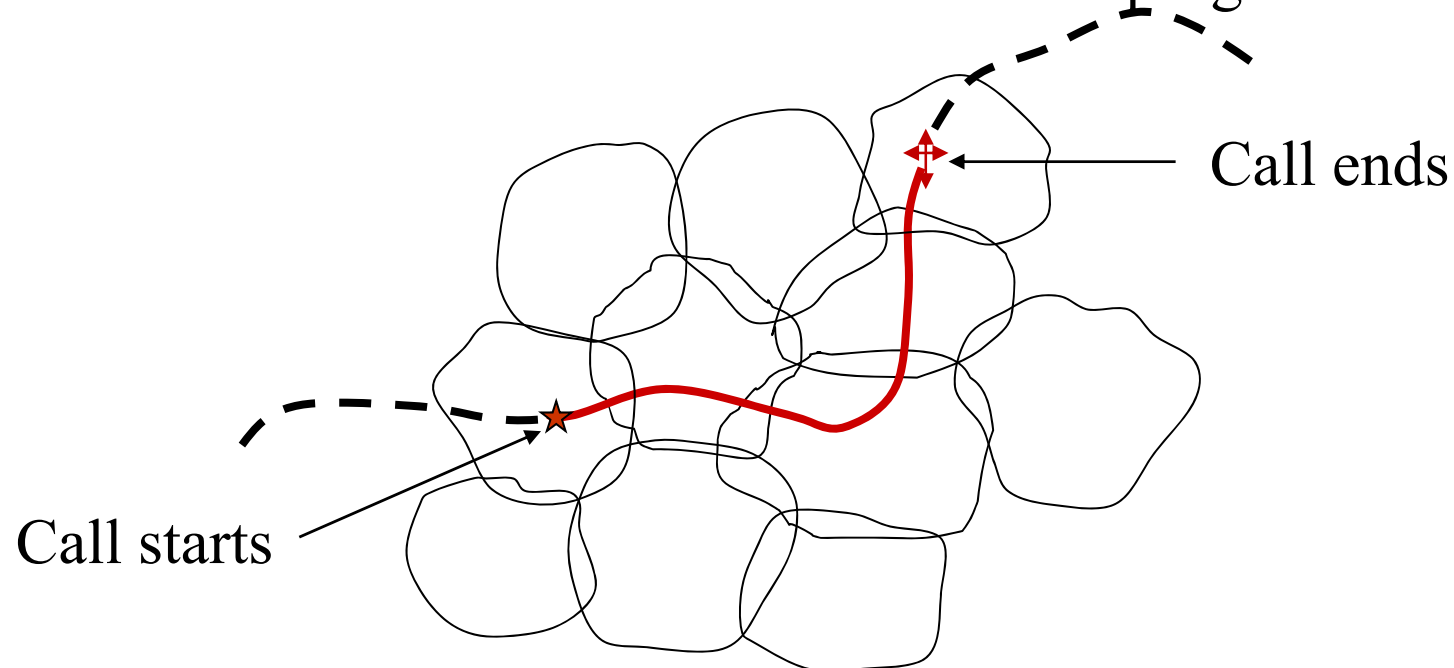$$f(t)_{TR} = \mu_R \cdot \exp(-\mu_R.t)$$
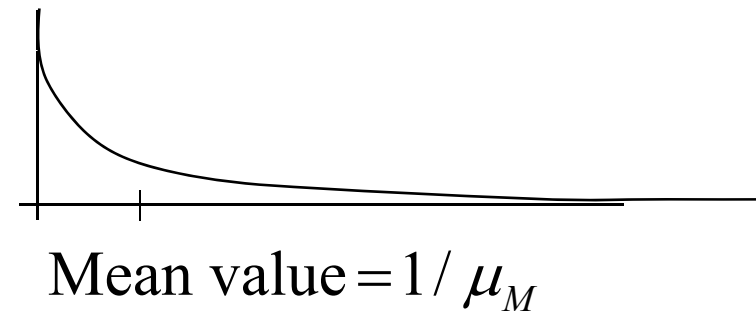
Trajectory

Mean value $= 1/\mu_R$

Call holding time

- - - - - Mobile terminal in idle state (attached)
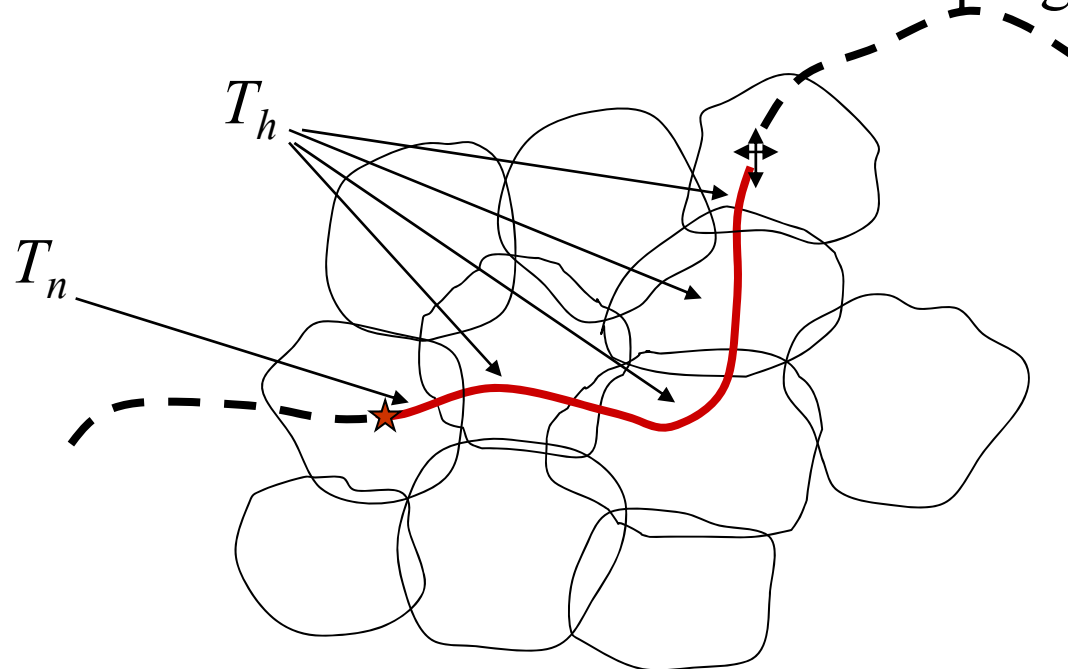
———— Mobile terminal with a call in progress

Call ends

Call starts

T_M = Call or Message holding time

$f(t)_{TM} = \mu_M \cdot \exp(-\mu_M . t)$

Mean value $= 1 / \mu_M$

Channel holding time

- – – – – Mobile terminal in idle state (attached)

- —————— Mobile terminal with a call in progress

T_h

T_n

T_n = Cell residence time, starting in an arbitrary point in the cell

T_h = Cell residence time, starting in an arbitrary point in the edge of the cell

$T_{Hn} = \min(T_M, T_n)$

$T_{Hh} = \min(T_M, T_h)$

$z = \min(x, y) \implies F_z(t) = F_x(t) + F_y(t) - F_x(t) \cdot F_y(t)$
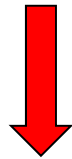
Cellular system as a queuing network

Memoryless model (MacMillan, -ITC: 1991)

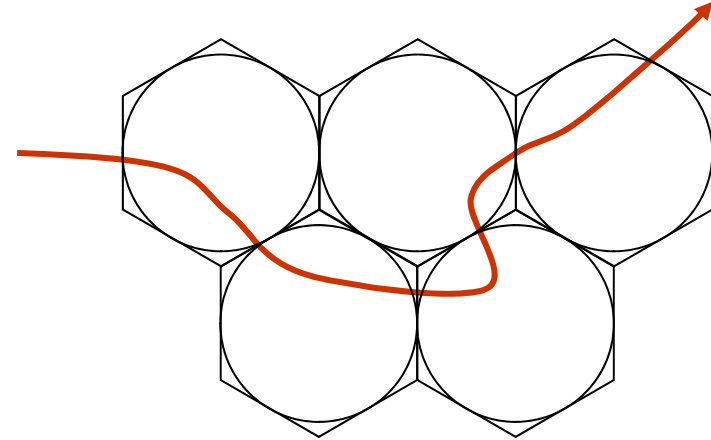T_R = Cell residence time

$$f(t)_{TR} = \mu_R \cdot \exp(-\mu_R.t)$$

T_M = Call or Message holding time

$$f(t)_{TM} = \mu_M \cdot \exp(-\mu_M.t)$$

T_{Ch} = Channel holding time

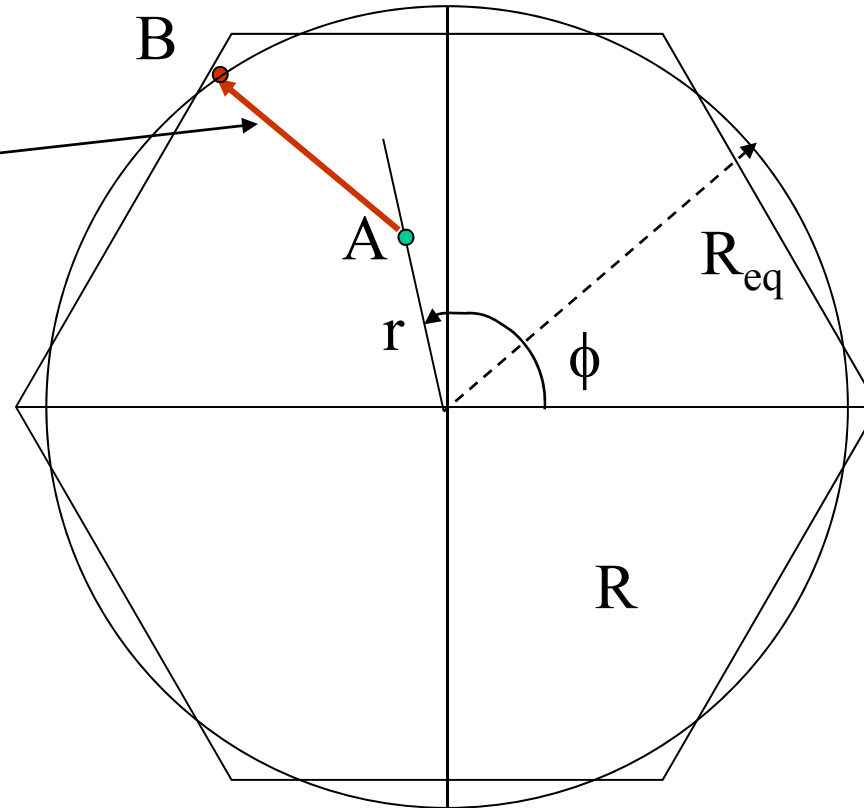$$f(t)_{T_{CH}} = \mu_{CH} \cdot \exp(-\mu_{CH}.t) = (\mu_R + \mu_M)\exp[-(\mu_R + \mu_M).t]$$

New cell residence time T_n

A: starting point
B: ending point

(From Hong, Rappaport -1986)

Trajectory

B

A

r

ϕ

R_{eq}

R

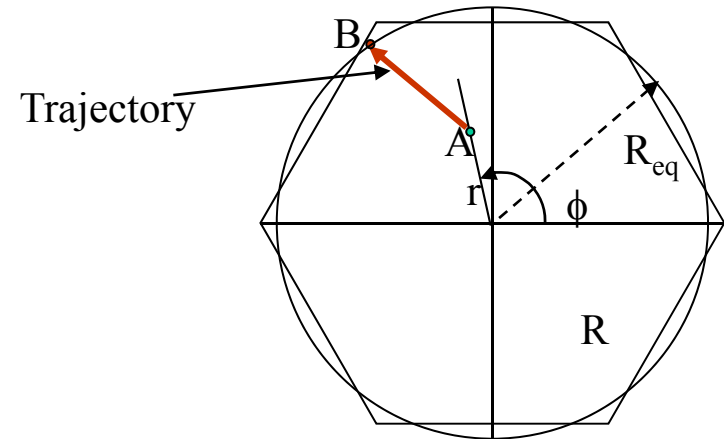Random initial direction, uniformly distributed in the interval $[0, 2\pi]$

Random initial speed, uniformly distributed in the interval $[0, V_{max}]$

Cell residence time T_n

Hong, Rappaport –(1986)

$$F_{Tn}(t) = \begin{cases} \frac{2}{\pi}\arcsin\left(\frac{V_{max}t}{2R_{eq}}\right) - \frac{4}{3\pi}\tan\left[\frac{1}{2}\arcsin\left(\frac{V_{max}t}{2R_{eq}}\right)\right] \\ +\frac{1}{3\pi}\left[2\arcsin\left(\frac{V_{max}t}{2R_{eq}}\right)\right]; \quad \text{for } 0 \le t \le \frac{2R_{eq}}{V_{max}}. \\[3mm] 1 - \frac{8R_{eq}}{3\pi V_{max}}\frac{1}{t}; \qquad\qquad \text{for } t > \frac{2R_{eq}}{V_{max}}. \end{cases}$$

Trajectory

Xie, Goodman –(1993) and Yeung, Nanda –(1996) , the "biased sampling problem"

$$F_X(x) = \begin{cases} \frac{2}{\pi R_{eq}^2}\left[\frac{x}{2}\sqrt{R_{eq}^2 - \left(\frac{x}{2}\right)^2} + R_{eq}^2\arcsin\left(\frac{x}{2R_{eq}}\right)\right]; \quad \text{for } 0 \le t \le 2R_{eq} \\ 1; \qquad\qquad\qquad\qquad\qquad\qquad \text{elsewhere} \end{cases}$$

$$F_{T_n}(t) = \int_0^t f_{T_n}(s)ds = \int_0^t\int_0^{2R_{eq}/s}\frac{2v}{\pi R_{eq}^2}\sqrt{R_{eq}^2 - \left(\frac{sv}{2}\right)^2}f_V(s)dvds; \quad \text{for } t \ge 0$$

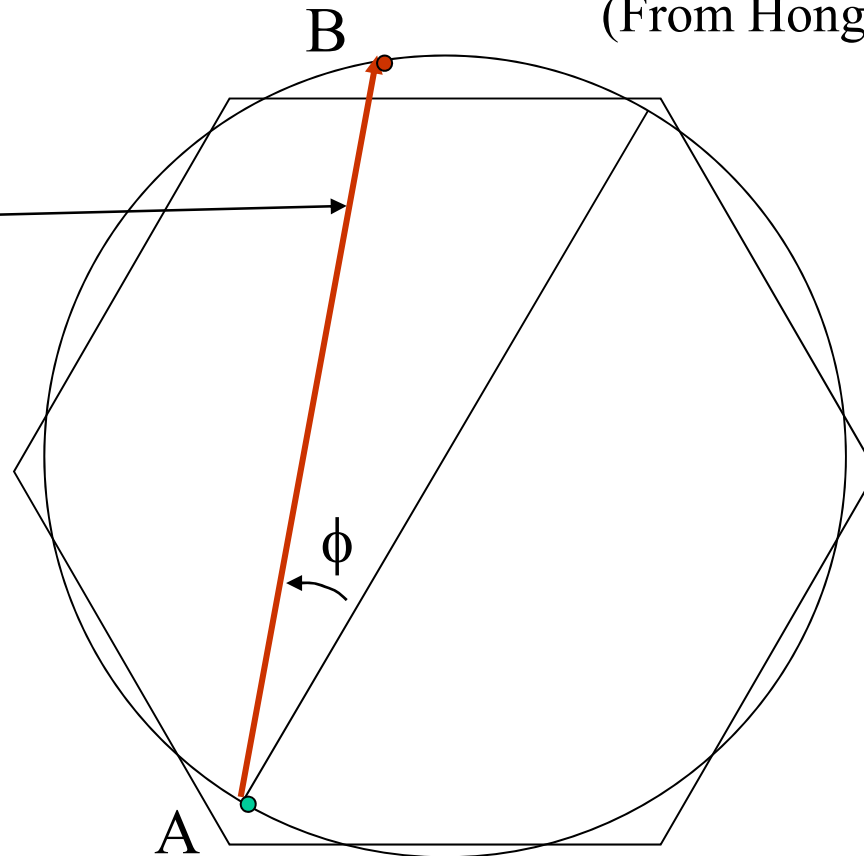$$E[T_n] = \int_0^\infty tf_{T_n}(t)dt = \frac{8R_{eq}^3}{3v}E[\frac{1}{V}]$$

Handover cell residence time T_h

A: starting point
B: ending point

B

(From Hong, Rappaport -1986)

Trajectory

ϕ

A

Random initial direction, uniformly distributed in the interval $[-\pi, \pi]$
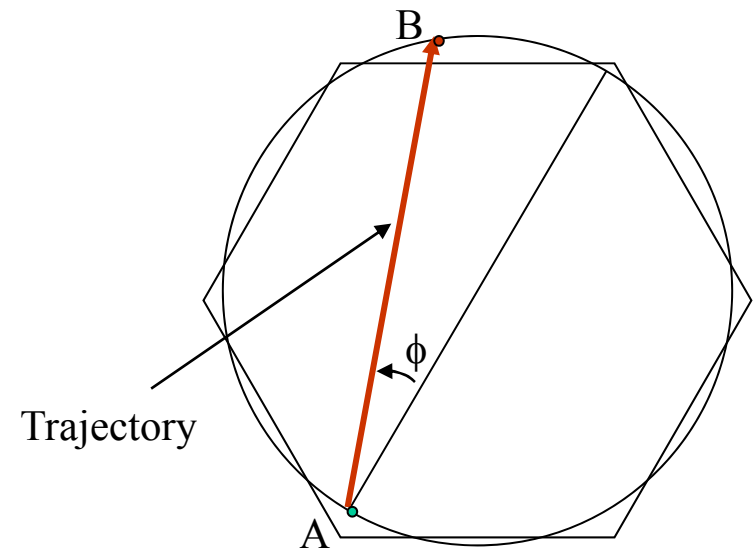
Random initial speed, uniformly distributed in the interval $[0, V_{max}]$

Cell residence time T_h

From Hong, Rappaport -1986)

$$F_{Th}(t) = \begin{cases} 0, & \text{for } t < 0 \\[2mm] \dfrac{2}{\pi}\arcsin\left(\dfrac{V_{max}t}{2R_{eq}}\right) - \dfrac{2}{\pi}\tan\left[\dfrac{1}{2}\arcsin\left(\dfrac{V_{max}t}{2R_{eq}}\right)\right] & \\[2mm] \qquad \text{for } 0 \le t \le \dfrac{2R_{eq}}{V_{max}} & \\[2mm] 1 - \dfrac{4R_{eq}}{\pi V_{max}}\dfrac{1}{t}, \text{ for } t > \dfrac{2R_{eq}}{V_{max}}. & \end{cases}$$

Trajectory

Xie, Goodman –(1993) and Yeung, Nanda –(1996) , the "biased sampling problem"

$$F_X(x) = \begin{cases} 1 - \dfrac{\sqrt{4R_{eq}^2 - y^2}}{2R_{eq}}; & \text{for } 0 \le y \le 2R_{eq} \\[2mm] 1; & \text{elsewhere} \end{cases}$$

$$F_{T_y}(t) = \int_0^t f_{T_y}(s)\,ds = \int_0^t \int_0^{2R_{eq}/s} \frac{1}{2R_{eq}E[V]}\frac{v^3 s}{2R_{eq}\sqrt{4R_{eq}^2 - (sv)^2}} f_V(s)\,dv\,ds; \quad \text{for } t \ge 0$$

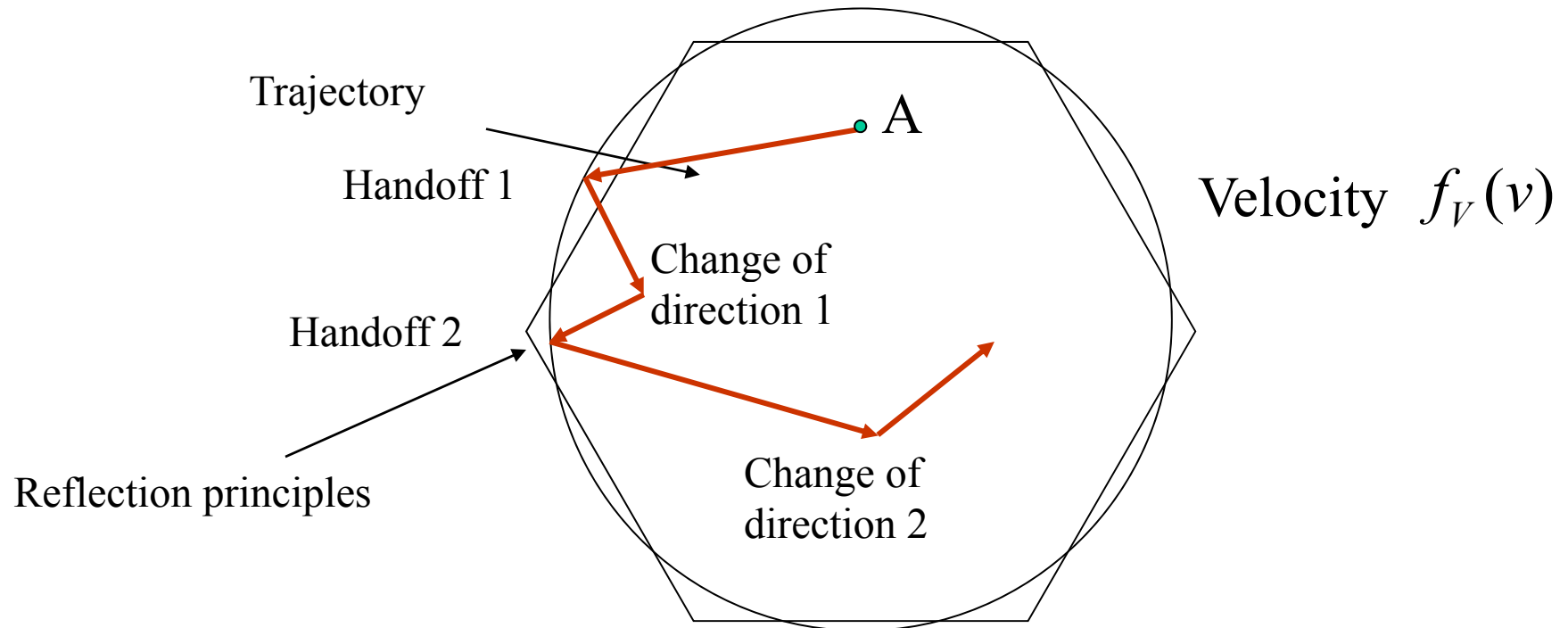$$E[T_h] = \int_0^\infty t f_{T_h}(t)\,dt = \frac{\pi}{2E[V]}$$

Cell - channel residence time T_c

Trajectory

Handoff 1

Change of direction 1

Handoff 2

Velocity $f_V(v)$

A

Reflection principles

Change of direction 2

Arbitrary initial point with direction uniformly distributed in $[0, 2\pi]$

Change of direction, uniformly distributed in the interval $[0, 2\pi]$

Time between changes of directions: distribution $f_T(t) = \mu e^{-\mu t}$

This model has been considered as the foundation for a number of mobility models.

Cell residence time in square cells, T_{sq}

- In urban environments and for in-home networks, the irregular cell shape becomes more and more regular (shadowing effects, …)
- The cell residence time in a square cells and in a rectangular cell has been evaluated in (Scheweigel, Zhao- 2002), (Scheweigel, - ITC:2003)
- Exponential approximation has been proposed.

A: starting point
B: ending point

New calls

Handover calls
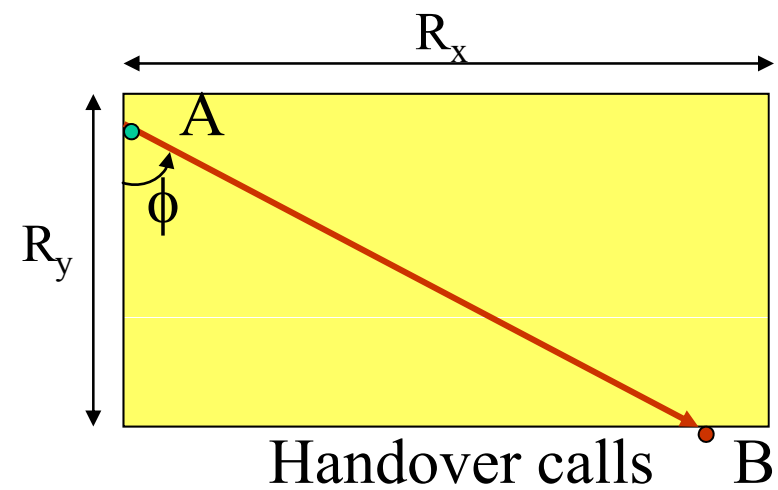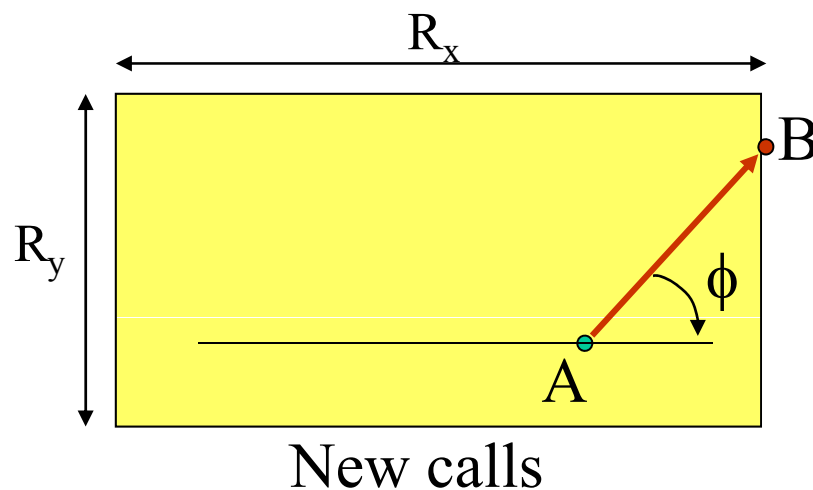
Fitting distributions -i-

- Exponential distribution (Hong, Rappaport -1986)
  - Extremely simple, although acceptable for macrocells, but not for micro and pico-cells

$$\int_0^\infty (F^C{}_{TH}(t) - e^{-\mu_H t})d(t) = 0 \qquad G = \frac{\int_0^\infty | F^C{}_{TH}(t) - e^{-\mu_H t} | d(t)}{2\int_0^\infty F^C{}_{TH}(t)d(t)}$$

Fitting distributions -ii-

- ## Generalized gamma (Zonoozi & Dassanayake -1997)
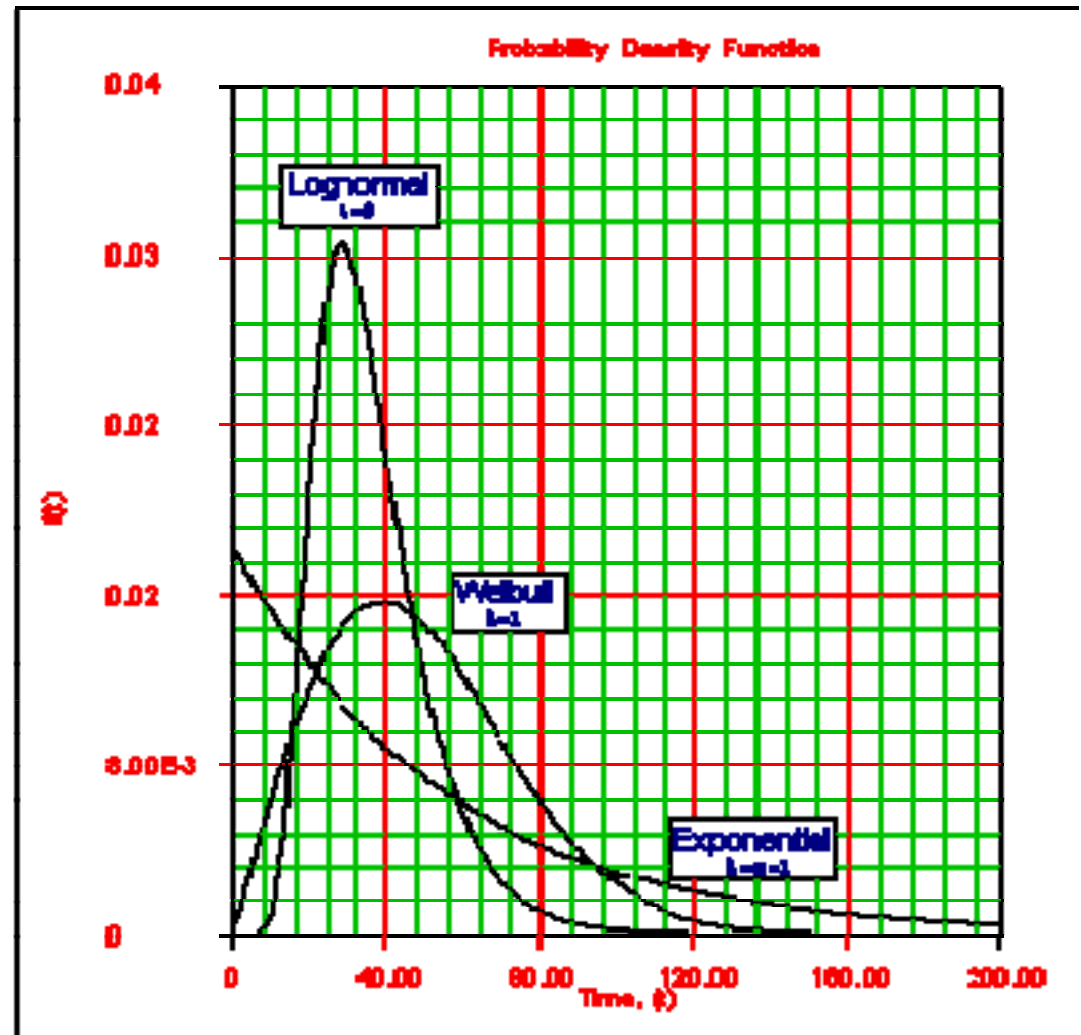 - Good enough for **macrocells**, but not for **micro** and **picocells**

$$f_{TC}(t; k, \beta, \theta) =$$

$$= \frac{\beta}{\theta\,\Gamma(k)} \left(\frac{t}{\theta}\right)^{(k\beta-1)} e^{-(t/\theta)^{\beta}}$$

$\theta > 0$ is a scale parameter, $\beta > 0$ and $k > 0$ are shape parameters

$$\mu = \ln(\theta) + \frac{1}{\beta} . \ln\left(\frac{1}{\lambda^2}\right)$$

$$\sigma = \frac{1}{\beta\sqrt{k}}$$

$$\lambda = \frac{1}{\sqrt{k}}$$

Fitting distributions -iii-

- "SOHYP" (Sum of hyper-exponentials) distribution (Olik, Rappaport -1998)

  - Quite simple, although acceptable for macrocells, but not for micro and picocells

$$f_{\exp}(t) = \sum_{i=1}^{M(1)} \alpha_{1i} m_{1i} \eta_{1i} e^{-m_{1i}\eta_{1i}t} * \sum_{i=1}^{M(2)} \alpha_{2i} m_{2i} \eta_{2i} e^{-m_{2i}\eta_{2i}.t} * ....$$

- Hyper-Erlang distributions (Fang & Chlamtac -1999)

  - Quite simple and good enough to fit field data

$$f_{herl}(t) = \sum_{i=1}^{M} \alpha_i \frac{(m_i \eta_i)^{m_i} t^{m_i-1}}{(m_i - 1)!} e^{-m_i\eta_i t}$$

Handover area residence time -i-

- A regular geometry was chosen in the modeling process
  - While in the handover area, no change in the movement (speed and direction) occurs.

A: starting point
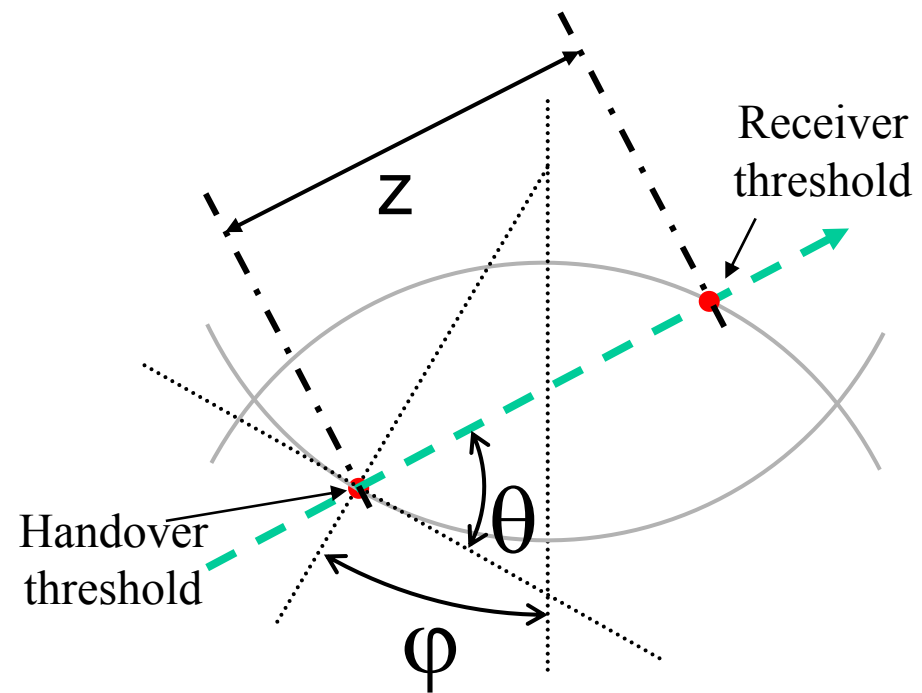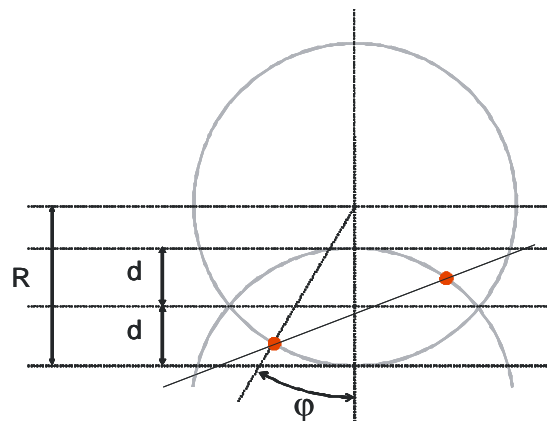B: ending point
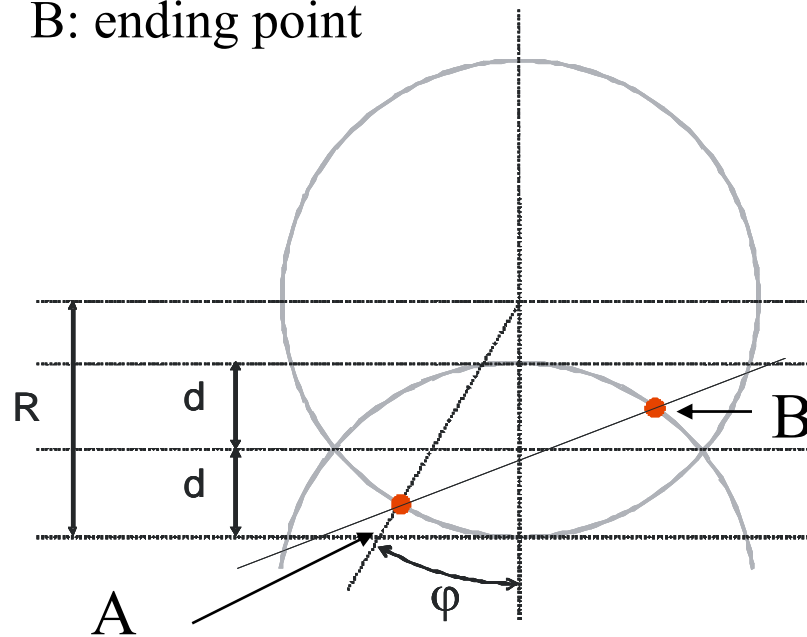
Handoff area

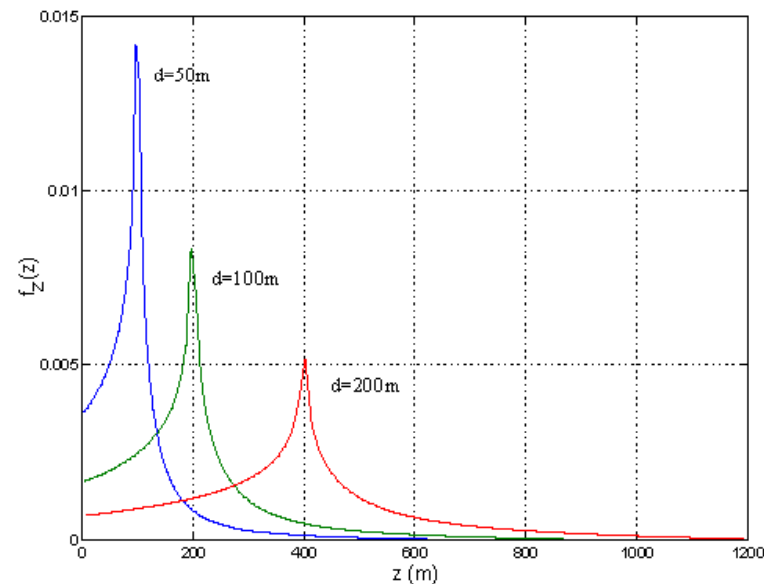Diagram of angles and domain of θ and φ

Handover area residence time -ii-

Moving distance and dwell time are related by → $T_d = \dfrac{Z}{V}$

Distribution of Z. A numerical version is obtained → $f_Z(Z)$

Distribution of V. A truncated Gaussian distribution → $f_V(V)$
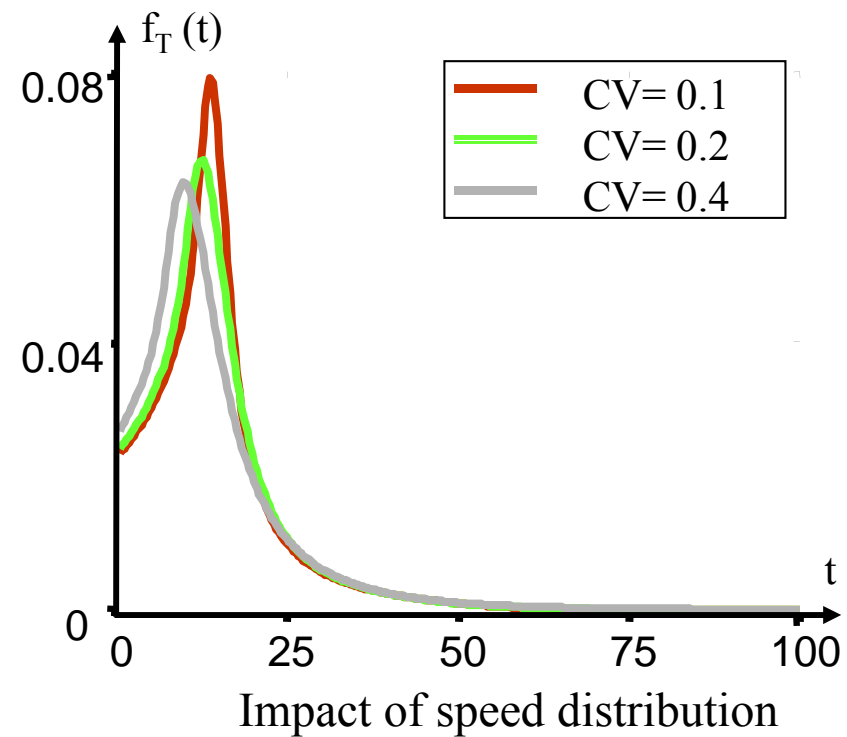

Handoff area
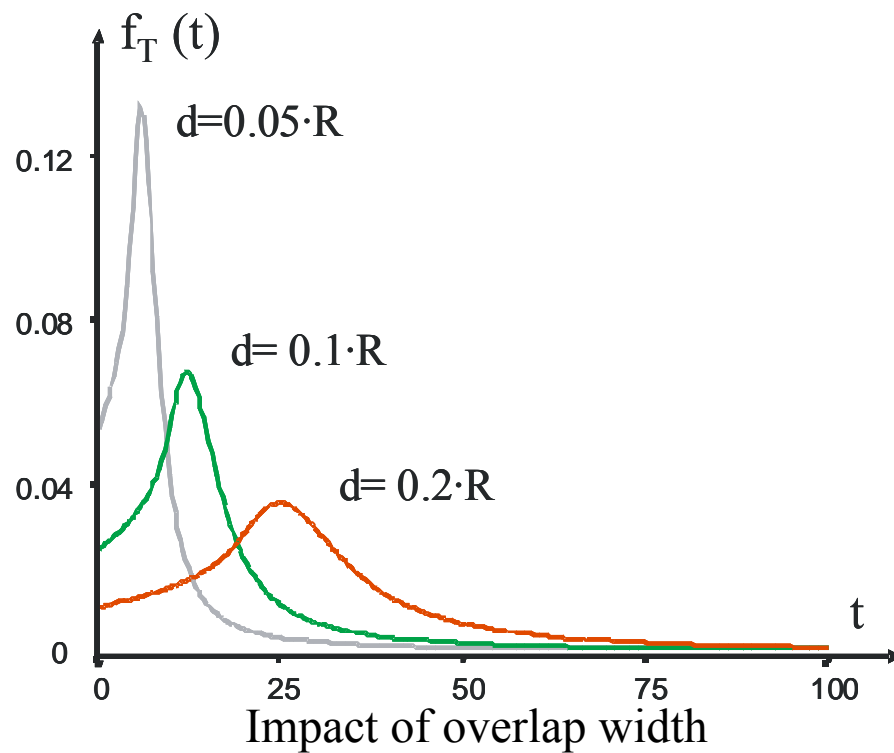

pdf of traveled distance

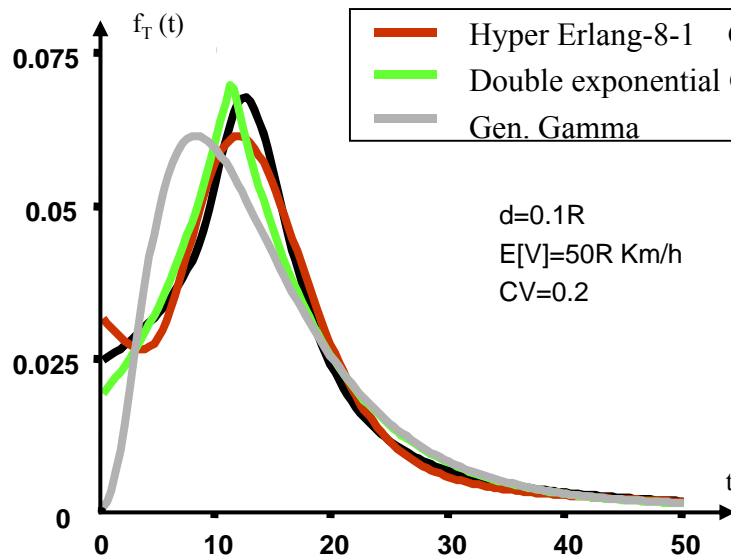# Handover area residence time -iii-

- Impact of overlap and speed distribution (Pla, Casares -2002):

Impact of overlap width

Impact of speed distribution

Handover area residence time -iv-

- Fitting procedure (Pla, Casares -2002):
  - Generalized gamma distribution
    (includes the gamma, lognormal, Weibull, as particular cases)
  - Erlang –j,k
  - Hyper Erlang –j,k
  - Double exponential
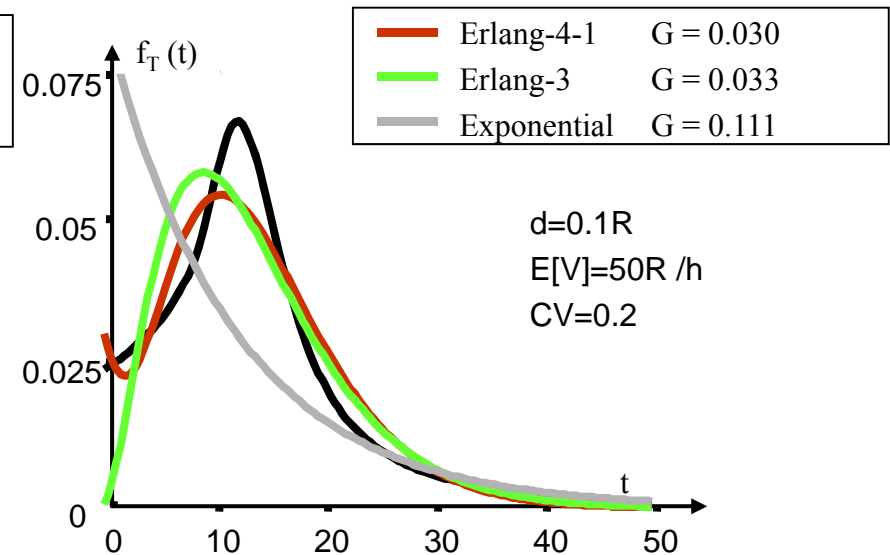
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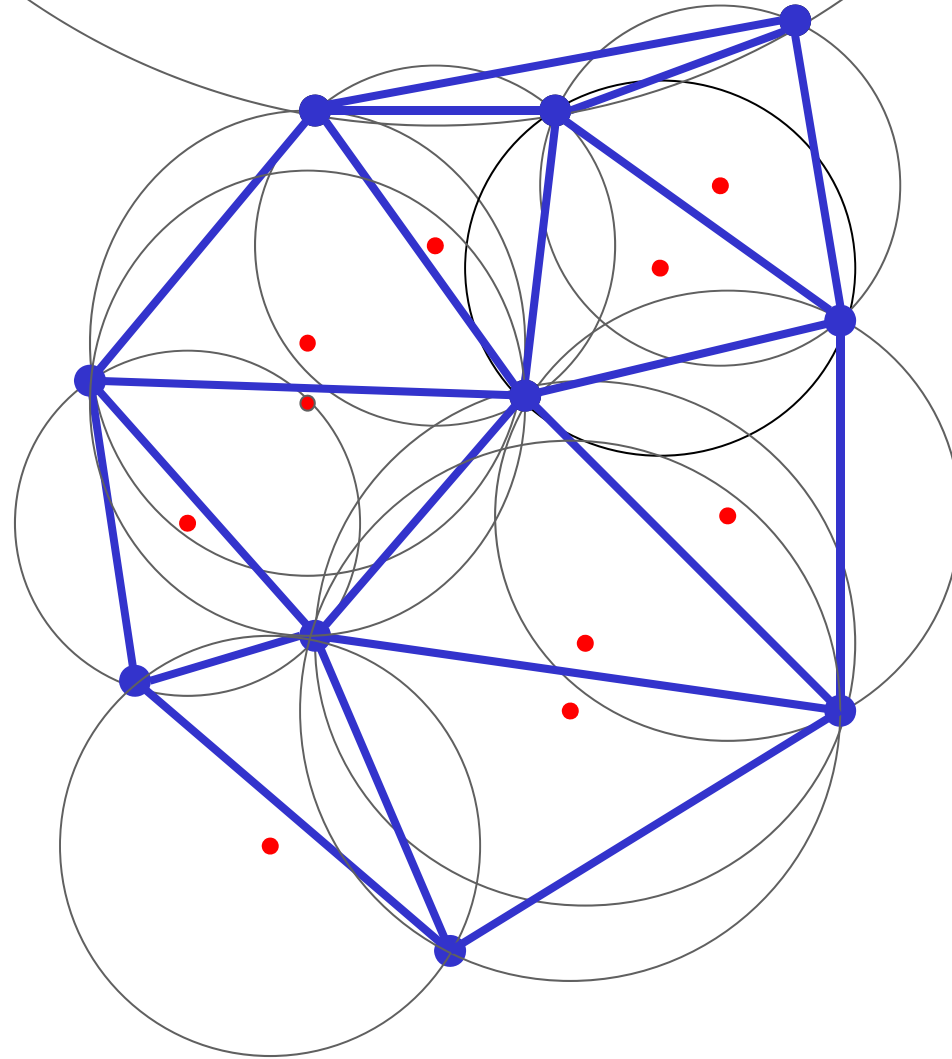
Tessellation of the plane
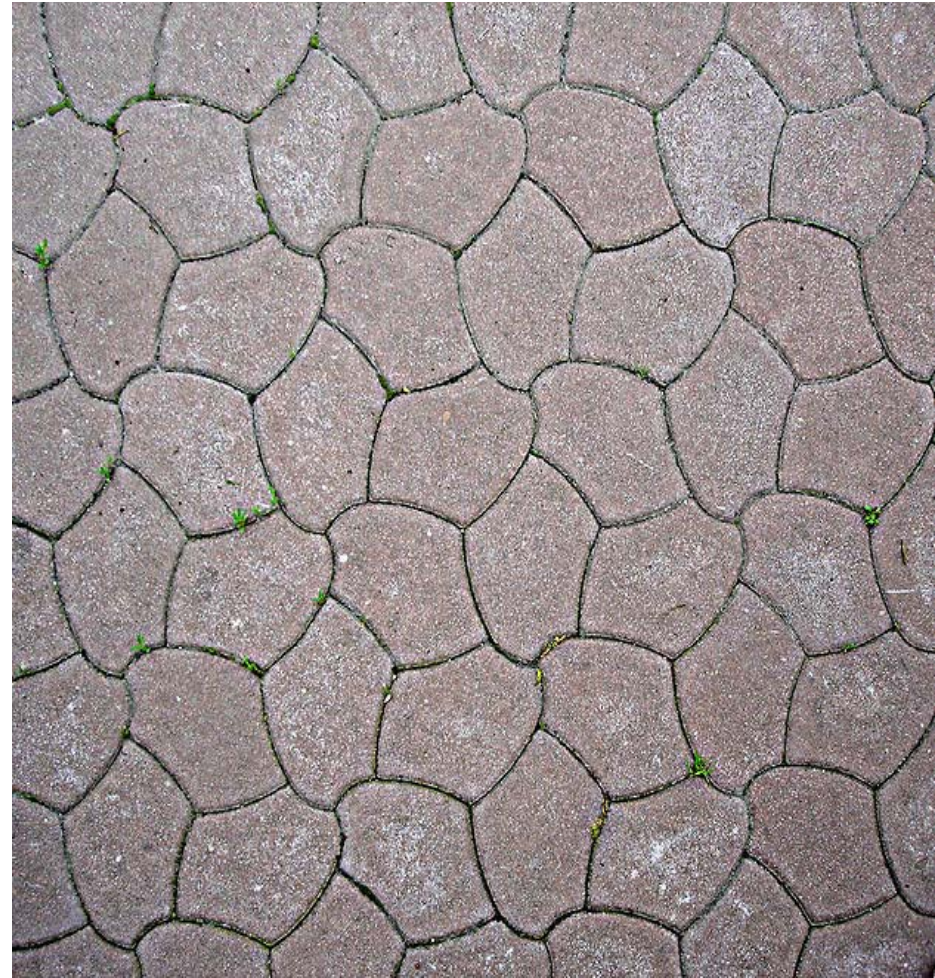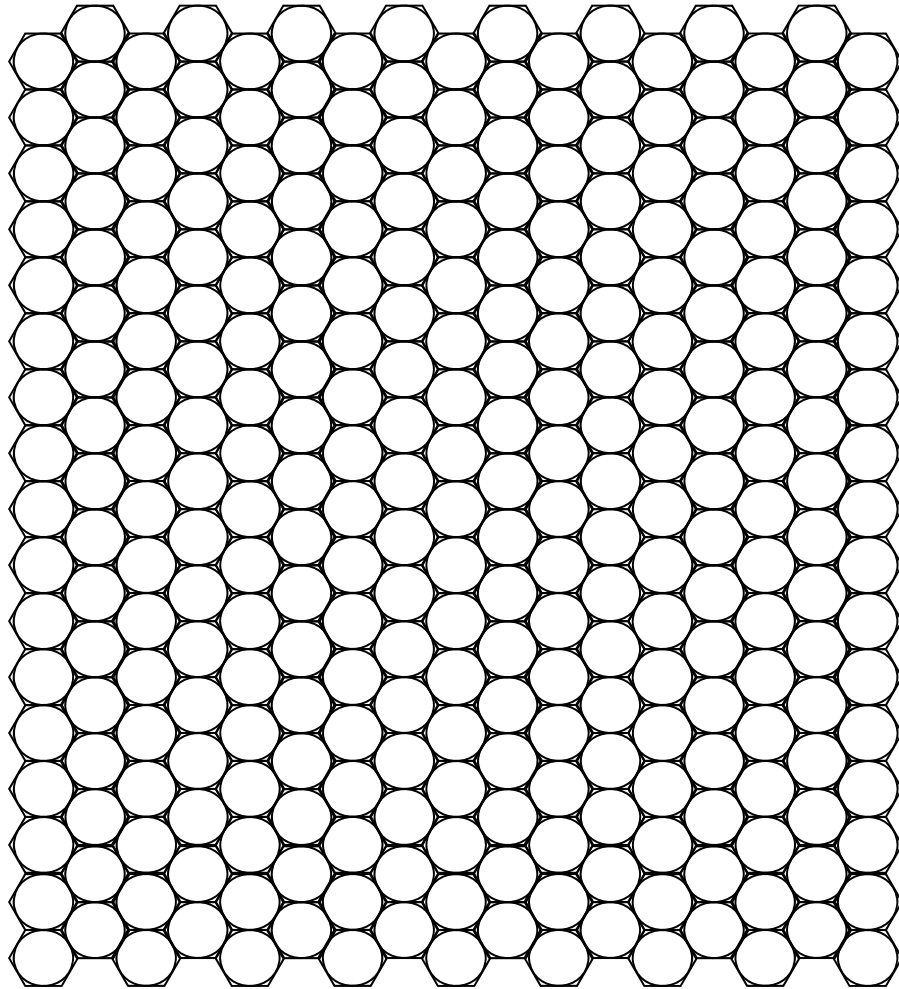
Delauney Tessellation

● Location Points
Cell sites

The **Delauney triangulation**, a dual model to **Voronoi tessellation**, is used as a (connected) graph of nearest neighbors (e.g. for routing purpose).

Delauney - Voronoi Tessellation

● Location Points:
Voronoi sites or
Voronoi points

Each site has a
Voronoi cell
A Voronoi cell is
a convex set

● Centers of the
circumferences:
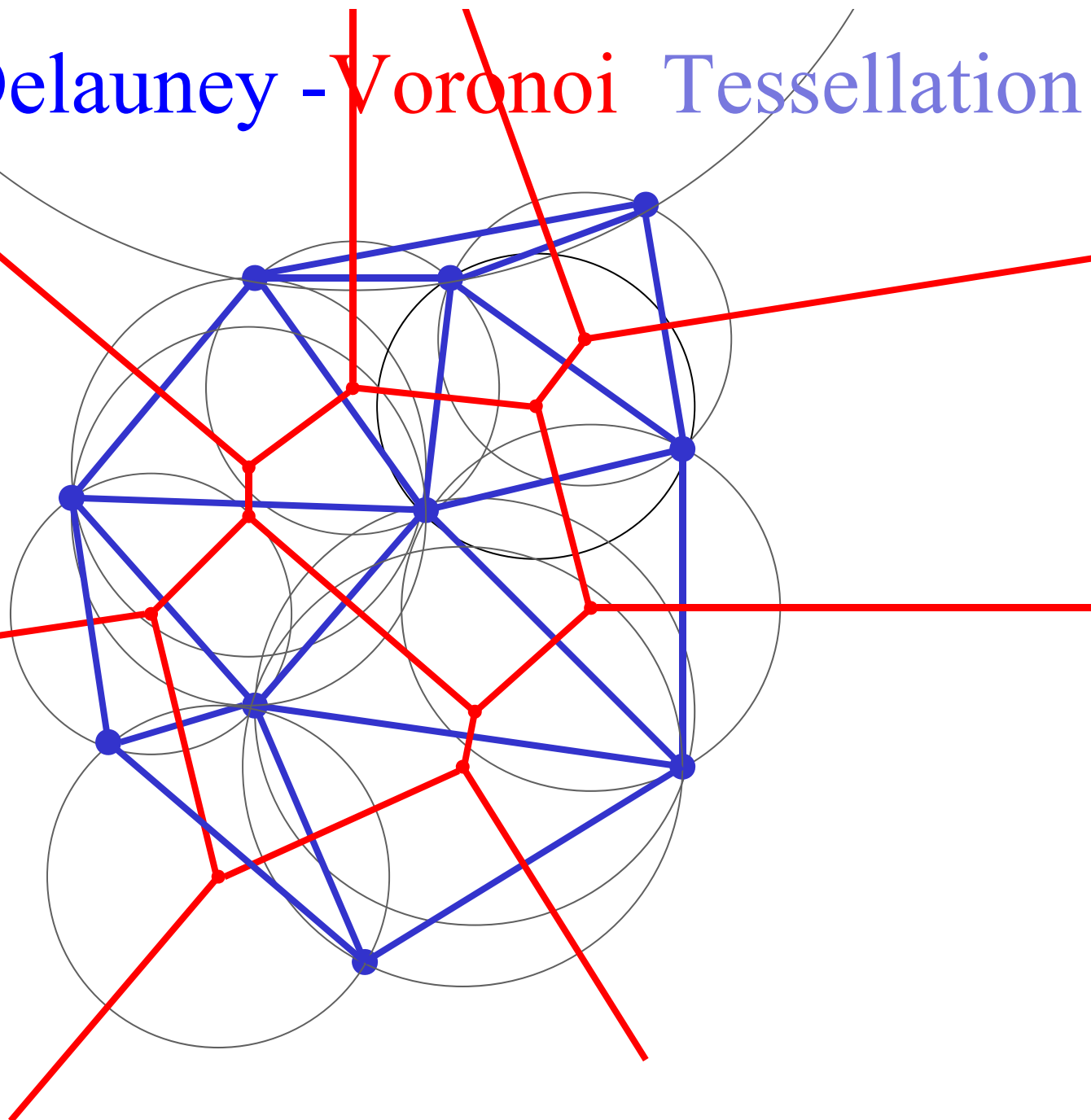Voronoi nodes
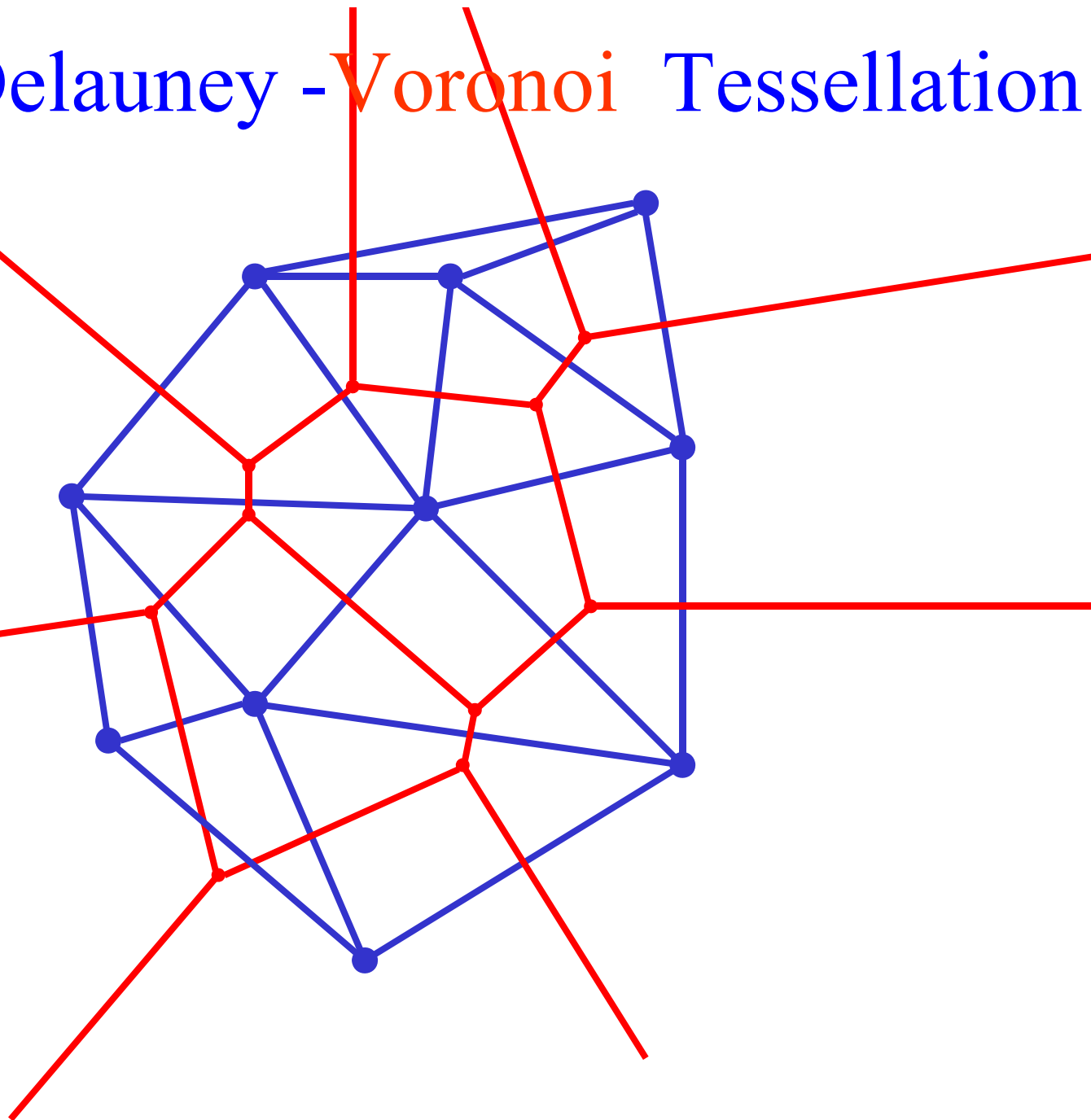
| Voronoi edges

Delauney - Voronoi Tessellation

- Location Points:
  Voronoi sites or
  Voronoi points

  Each site has a
  Voronoi cell
  A Voronoi cell is
  a convex set

- Centers of the
  circumferences:
  Voronoi nodes

  Voronoi edges

Voronoi Tessellation

- **Location Points**
  Cell sites:
  Voronoi sites or
  Voronoi points

  Each site has a
  Voronoi cell
  A Voronoi cell is
  a convex set
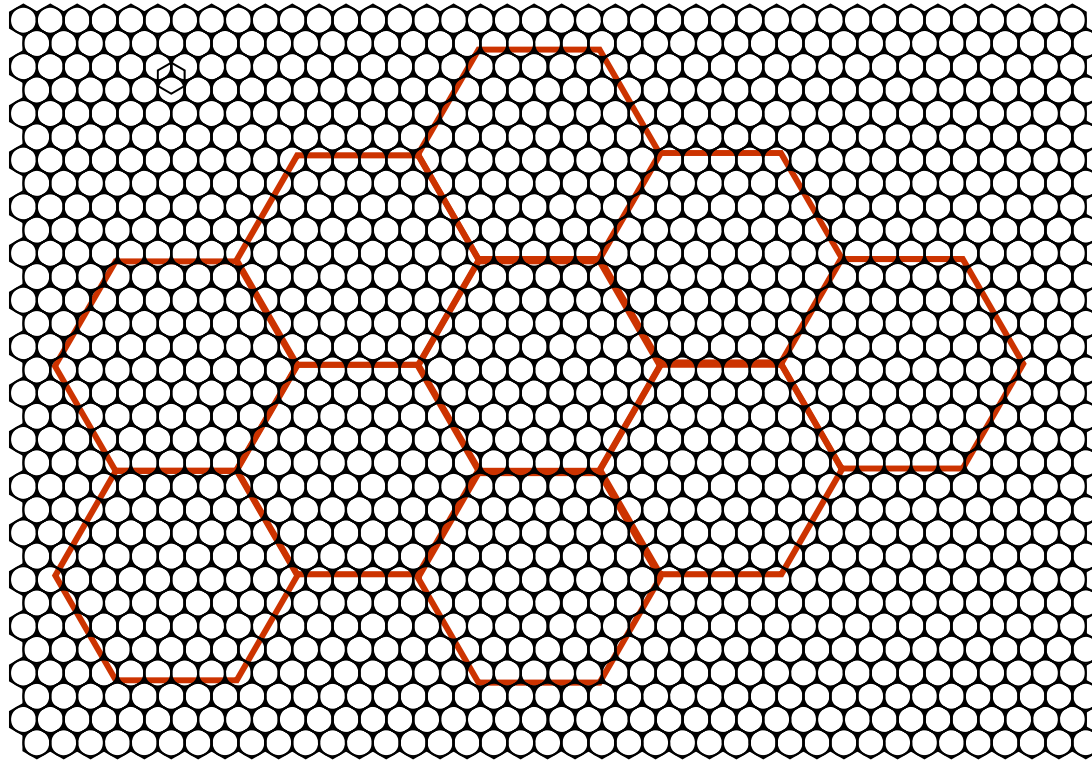
- **Voronoi nodes**

| **Voronoi edges**

If the blue points is a Poisson process we have a Poisson-Voronoi tessellation (PVT).
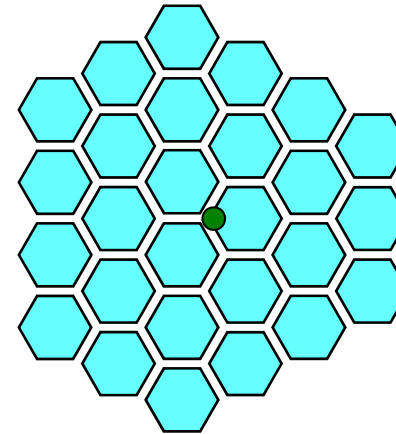
Location Areas

- Each location area (LA) contains a group of neighboring cells
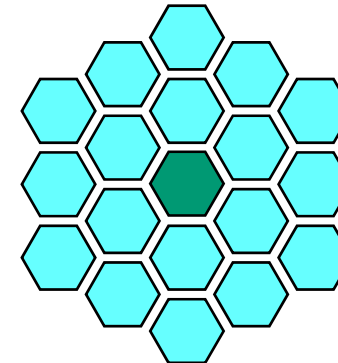
Mosaic Graphs

- Mosaic M:
  - (A) Rings of cells around an initial element: a point

(A): M_3

- Mosaic T:
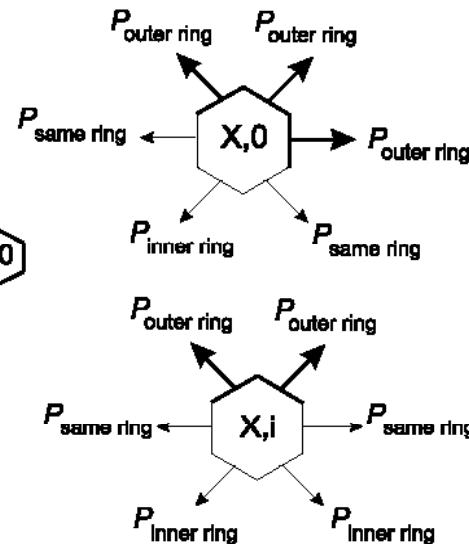 - (B) Rings of cells around an initial element: a cell

(B): T_2

- **Location area residence time**
  - If cell sojourn time is exponential: **Phase Type** distribution

Hexagonal scenario for MM

From Martínez, García, Casares - (2005, 2008)
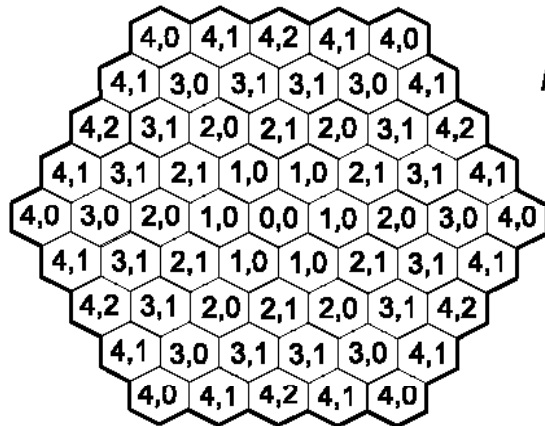
$$P_{\text{outer ring}} = \frac{\alpha}{3(1+\alpha)}$$

$$P_{\text{same ring}} = \frac{1}{3(1+\alpha)}$$

$$P_{\text{inner ring}} = \frac{1}{3(1+\alpha)}$$

$$P_{\text{outer ring}} = \frac{\alpha}{2(2+\alpha)}$$

$$P_{\text{same ring}} = \frac{1}{2(2+\alpha)}$$

$$P_{\text{inner ring}} = \frac{1}{2(2+\alpha)}$$

- Directional movement parameter (α) values within $[0,\infty[$

  - $0 \leq \alpha < 1$: High probability of moving towards an inner ring or being roaming within the same ring

  - $\alpha = 1$: Random walk mobility model

  - $1 < \alpha < \infty$: High probability of moving towards an outer ring

2-D conversion into 1-D
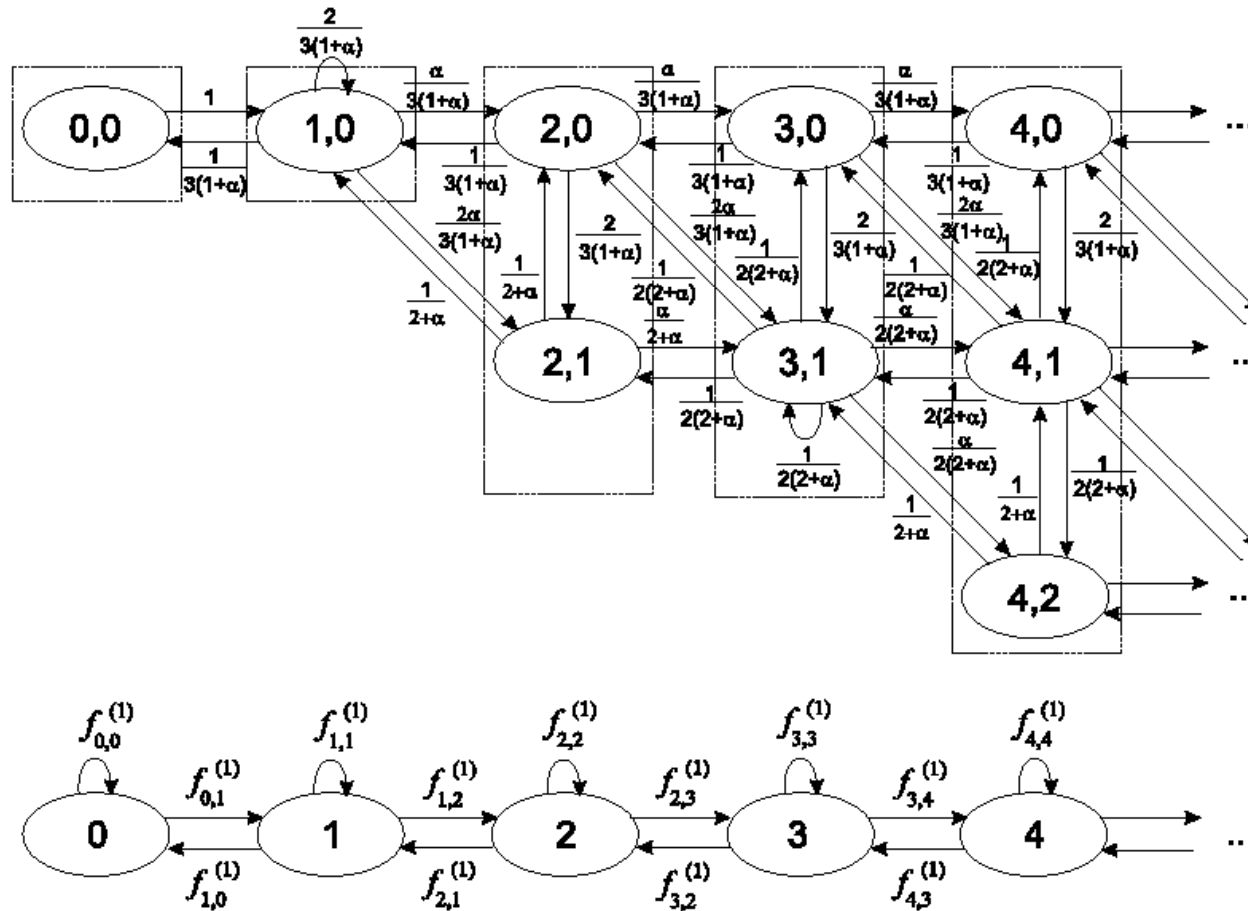
Cells are grouped by rings to obtain a 1D Markov chain that simplifies the model

Memoryless mobility model -

$$p_0 = P_0(\beta) = \frac{\beta^3}{D(\beta)}$$

$$p_1 = P_1(\beta) = p_5 = P_5(\beta) = \frac{\beta^2}{D(\beta)}$$

$$p_2 = P_2(\beta) = p_4 = P_4(\beta) = \frac{\beta}{D(\beta)}$$

$$p_3 = P_3(\beta) = \frac{1}{D(\beta)}$$

$$D(\beta) = \beta^3 + 2\beta^2 + 2\beta + 1$$

Random walk, when $\beta = 1$
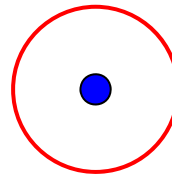
Straight ahead trajectory when $\beta \to \infty$
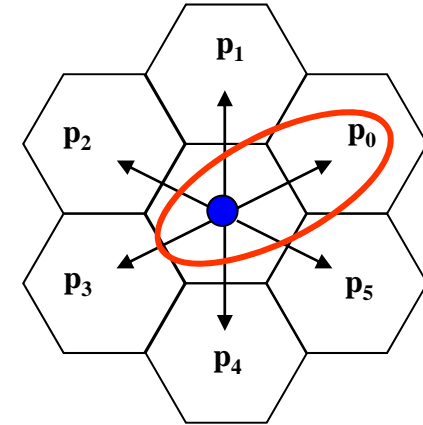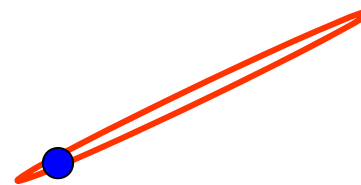
1-D Brownian motion –i-

Brownian motion is a particular case of random walk (Rose, Yates - 1997)

$$1 - p - q$$

$$q \qquad p$$

$$\Delta x$$
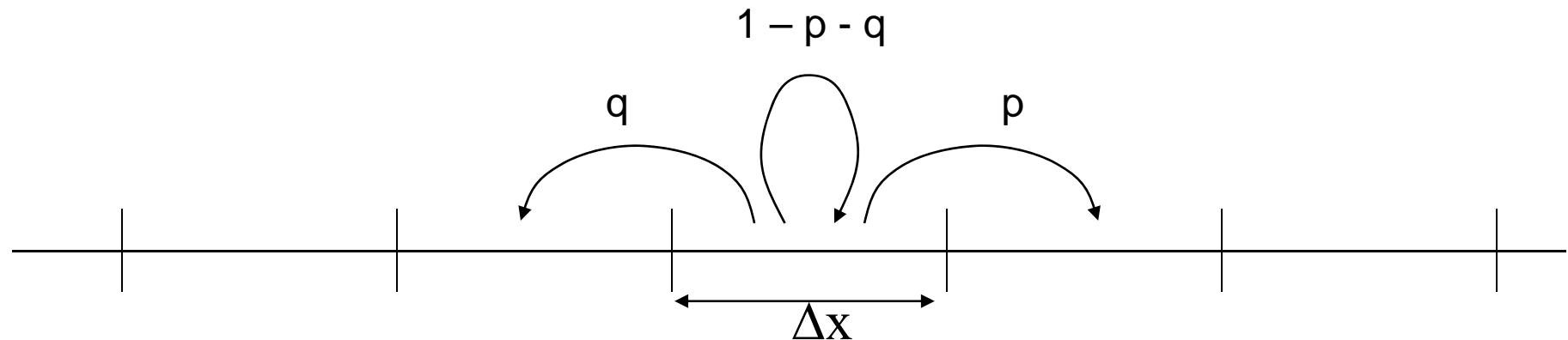
Drift velocity: $\qquad v = (p - q) \dfrac{\Delta x}{\Delta t}$

Diffusion constant: $\qquad D = 2 \left[(1 - p) \, p + (1 - q) \, q + 2 \, p \, q \right] \dfrac{(\Delta x)^2}{\Delta t}$

Probability density function: For $\Delta x, \Delta t$ very small; $\quad p(x, t) = \dfrac{1}{\sqrt{\pi D t}} e^{- \dfrac{(x - vt)^2}{Dt}}$

2-D Brownian motion –ii-

2-D: Personal location areas, recalculated after every contact with the system
(Rose, Yates - 1997)
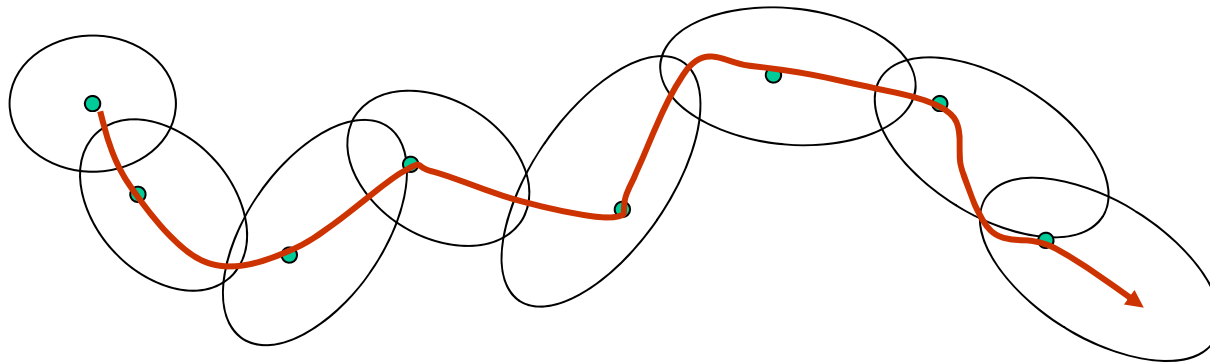
- ●   Contact with the system
- ⬭   Personal Location area

Probability density function:

$$z = x + jy = \sum_{k=1}^{M} a_k e^{j\theta_k} \qquad \text{when } M \to \infty \quad p(x,y) \to \frac{1}{4\pi \overline{A^2}} e^{-\frac{(x^2 + y^2)}{4\overline{A^2}}}$$

1-D Gauss-Markov

- Brownian motion can not represent the time correlation in mobile's velocity.

- Gaussian – Markov mobility model affords such a limitation (Liang, Haas -2003)

Autocorrelation function:

$$R_v = E[v(t)v(t+\tau)] = \sigma^2 e^{-\beta|t|} + \mu$$

Discrete Gauss-Markov process:

$$v_n = \alpha v_{n-1} + (1-\alpha)\mu + \sigma\sqrt{1-\alpha^2}\, w_{n-1}$$

with:

$$v_n = v(n\Delta t); \qquad \alpha = e^{-\beta\Delta t}$$

and $\{w_n\}$ uncorrelated Gaussian process with zero mean and unit variance, independent of $\{v_n\}$

How versatile is the Gauss-Markov?

- Autocorrelation function:

$$R_v = E[v(t)v(t+\tau)] = \sigma^2 e^{-\beta|t|} + \mu$$

- Discrete Gauss-Markov process:

$$v_n = \alpha v_{n-1} + (1-\alpha)\mu + \sigma\sqrt{1-\alpha^2}\, w_{n-1}$$

- Random walk with drift:

$$If\ \alpha \to 0;\ \ or\ \ \beta \to \infty;$$

$$v_n = \mu + \sigma w_{n-1}; \qquad R_v(k) = \mu^2$$

- Fluid flow with constant speed:

$$If\ \alpha \to 1;\ \ or\ \ \beta \to 0;$$

$$v_n = v_{n-1}; \qquad R_v(k) = \sigma^2 + \mu^2$$

Mobility Model for Mobility Management

OUTLINE

- General aspects: scenarios, cells, ….
- Residence time and call holding times.
  - *Cell and handoff areas.*
- MM in location management.
  - *Tessellation of the plane.*
  - *Random walk.*
  - *Brownian motion.*
  - *Gauss-Markov.*
- **Advanced synthetic mobility models**
- **Macroscopic vision. Transportation theory.**
  - *Fluid flow models.*
  - *Gravity models.*
  - *Rush hour versus busy hour.*
  - *Flow, Speed and Density.*

Advanced synthetic mobility models

- Hong & Rapaport's model (1986), Pla & Casares's model (2002), Scheweigel, - (2003), ... do not allow that the direction of the MT changes in the walk area.

- Guerin's model (1986) and Zonoozi & Dassayanake's model (1997) allow a random change of direction, anywhere, anytime. These models can be seen as a generalization of previous models.

  - Basically, two types of mobility models have been used in ad-hoc scenarios, VANET, ..:
    - Random Way Point (RWP)
    - Random Direction (RD)

RWP and RD mobility models –i-

• **RWP:**

$$P_0, \ \Theta_1, D_1, V_1 \longrightarrow P_1$$

$$P_1, \ \Theta_2, D_2, V_2 \longrightarrow P_2$$

$$P_2, \ \Theta_3, D_3, V_3 \longrightarrow P_3$$
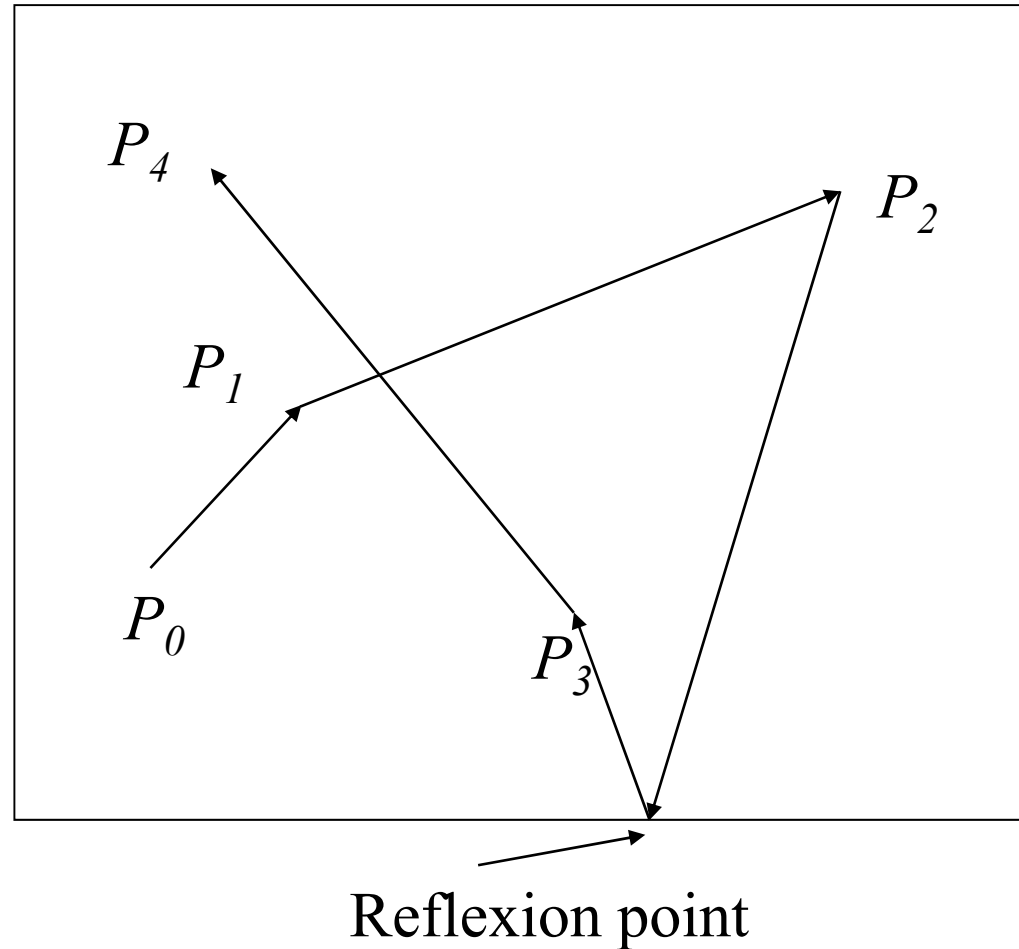
$$\cdots\cdots\cdots$$

• **RD:** As an alternative to overcome deficiencies of the RWP model

$$P_0, \ \Theta_1, T_1, V_1 \longrightarrow P_1$$

$$P_1, \ \Theta_2, T_2, V_2 \longrightarrow P_2$$

$$P_2, \ \Theta_3, T_3, V_3 \longrightarrow P_3$$

$$\cdots\cdots\cdots$$

Reflexion point

They can be extended by considering a pause interval between two consecutive trips

RWP and RD mobility models –ii-

- The Random Way Point Model suffers from its statistical properties. Node distribution does not stay constant on average during simulation time With ongoing time, node distribution tends towards the center of the simulation area and the mean value of the speed distribution decreases significantly

- In order to avoid this fatal behavior of the Random Waypoint Model, the RD mobility model was developed.
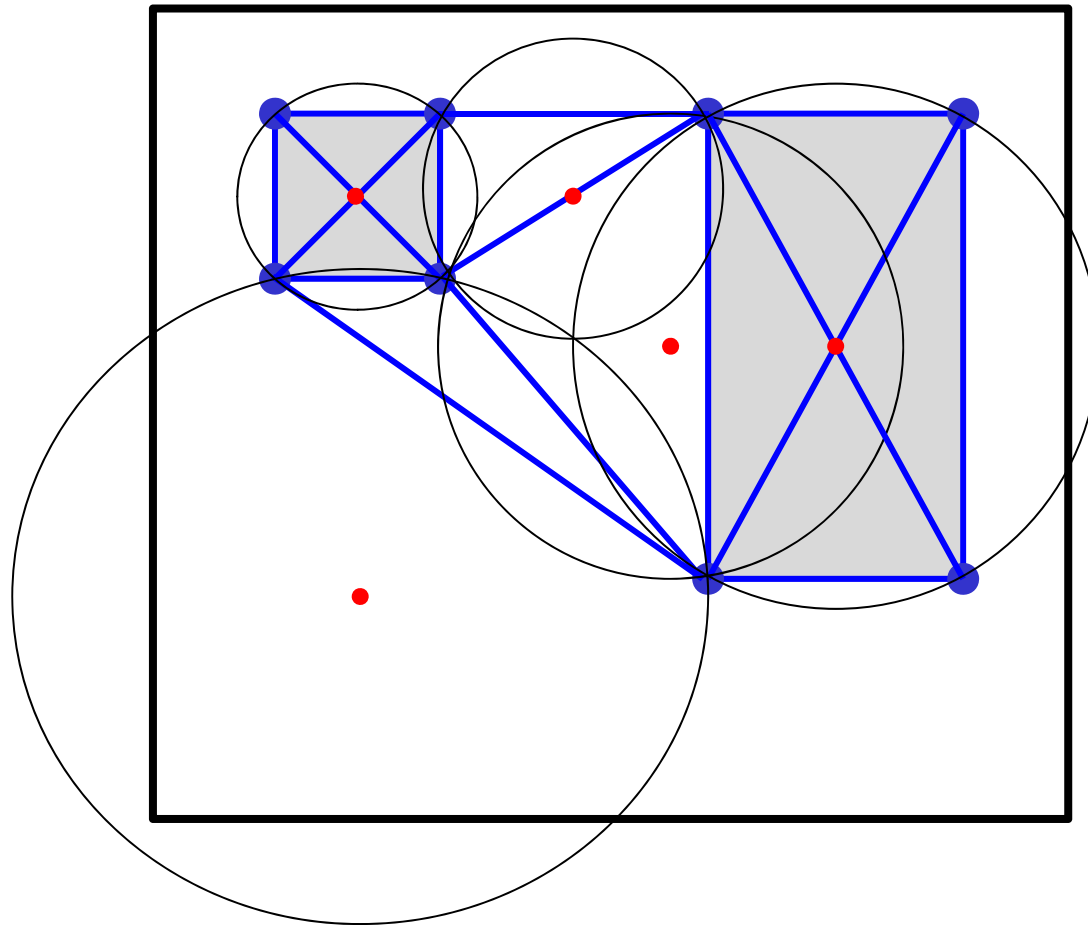
Obstacle Mobility Model –i-

- The Random Way Point Model suffers from its statistical properties. Node distribution does not stay constant on average during simulation time With ongoing time, node distribution tends towards the center of the simulation area and the mean value of the speed distribution decreases significantly

- In order to avoid this fatal behavior of the Random Waypoint Model, the RD mobility model was developed.

- But those models do not capture certain aspects of the reality, for instance: OBSTACLES

Obstacle Mobility Model

Corners of obstacles are sets of location points (not cell sites!)

Obstacle Mobility Model

- **Location points** (corners of obstacles)

Obstacle Mobility Model

- **Location points** (corners of obstacles)
- **Edges** of the Voronoi diagram
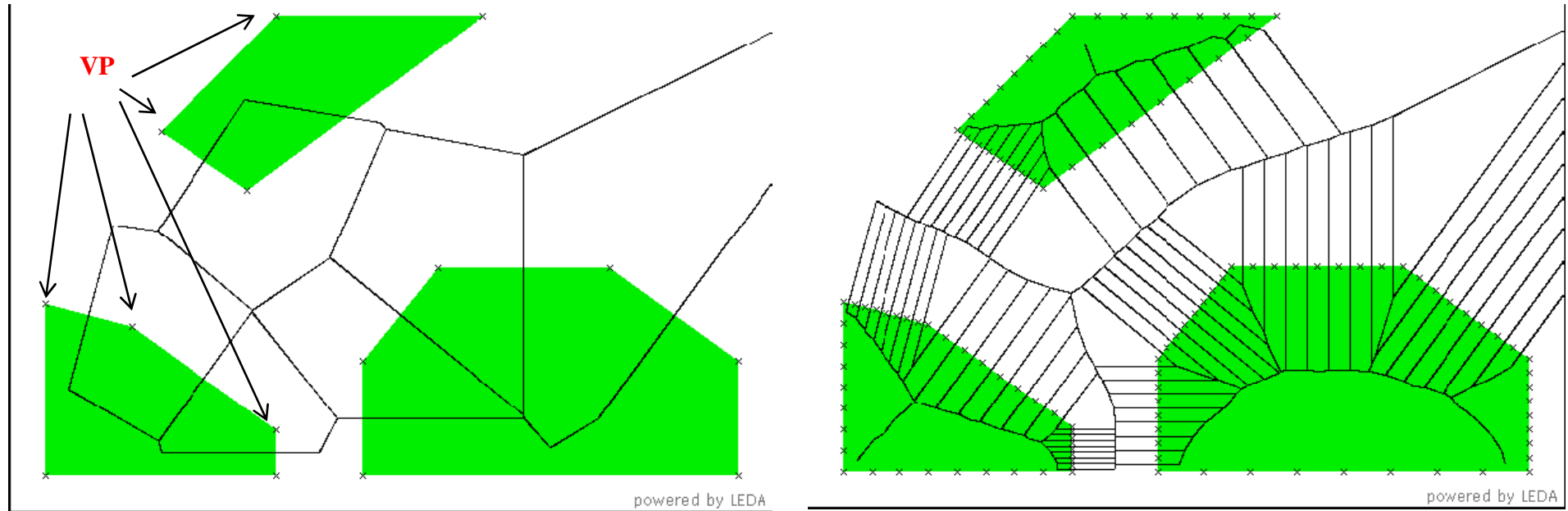- **Intersections** of the Voronoi graph with the borders
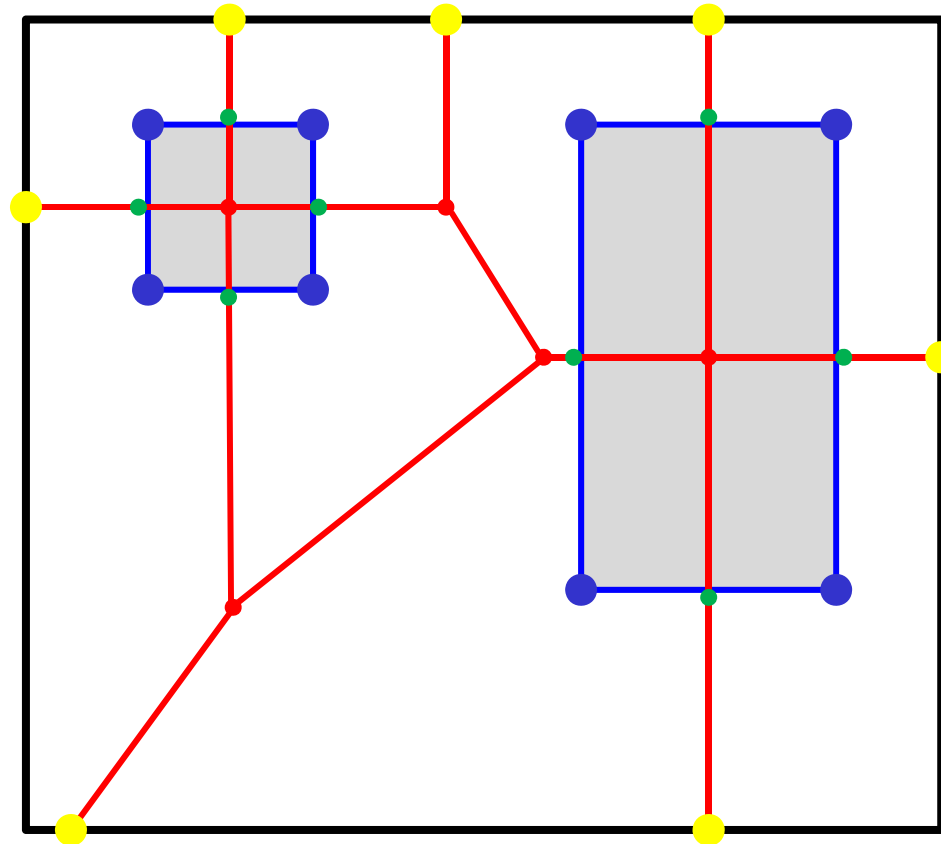
- **Doorways:** The points of intersections between the Voronoi graph and the obstacle boundaries

Obstacle Mobility Model

Jardosh (2003)
MobiCom's conference

Voronoi-based MM for urban environment –i-

Zimmermann, EWC'2005


•First of all, building structures are defined based upon a plain simulation area. Each building is uniquely defined by a given set of corner coordinates. In principle, arbitrary building shapes can thus be realized. The corners are used as Voronoi points for a first calculation of the Voronoi graph.

•Further interpolation points are used to get a finer structure of the Voronoi graph

Voronoi-based MM for urban environment –ii-

Zimmermann, EWC'2005
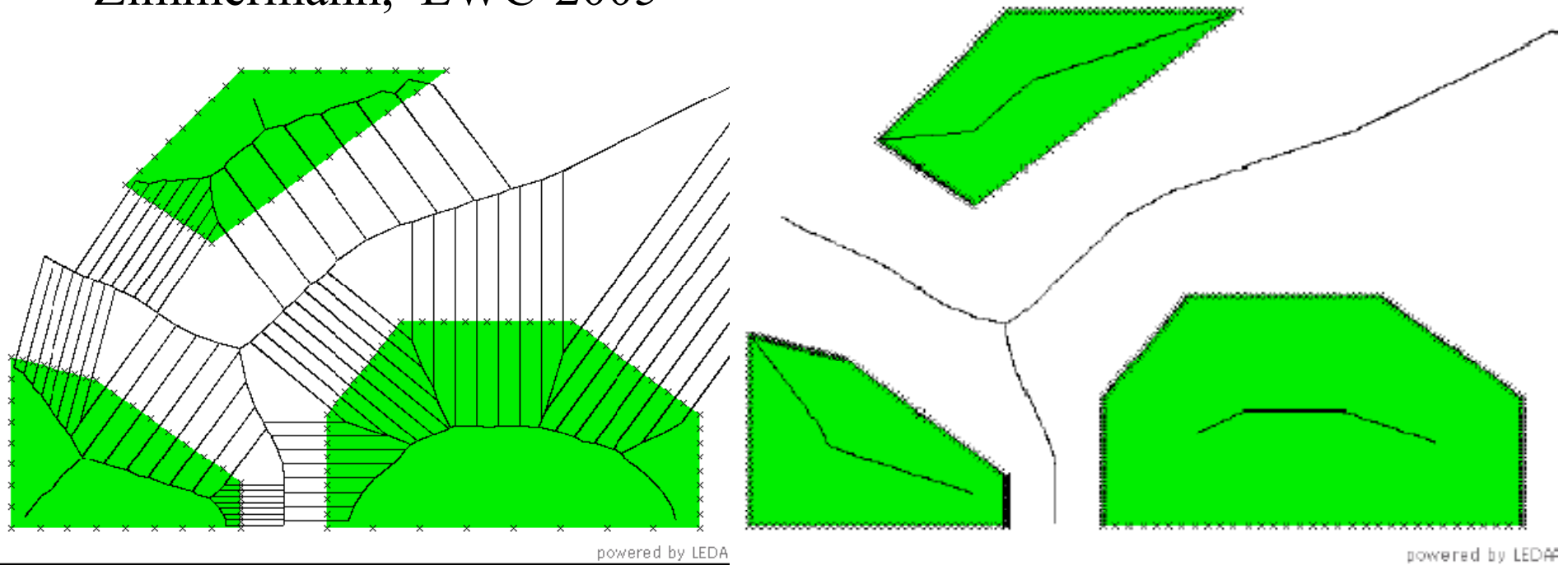
- A second Voronoi graph can be calculated based upon the merged structure between initial building structure and the first Voronoi graph.
- Further interpolation points are used to get a finer structure of the Voronoi graph

Voronoi-based MM for urban environment –iii-
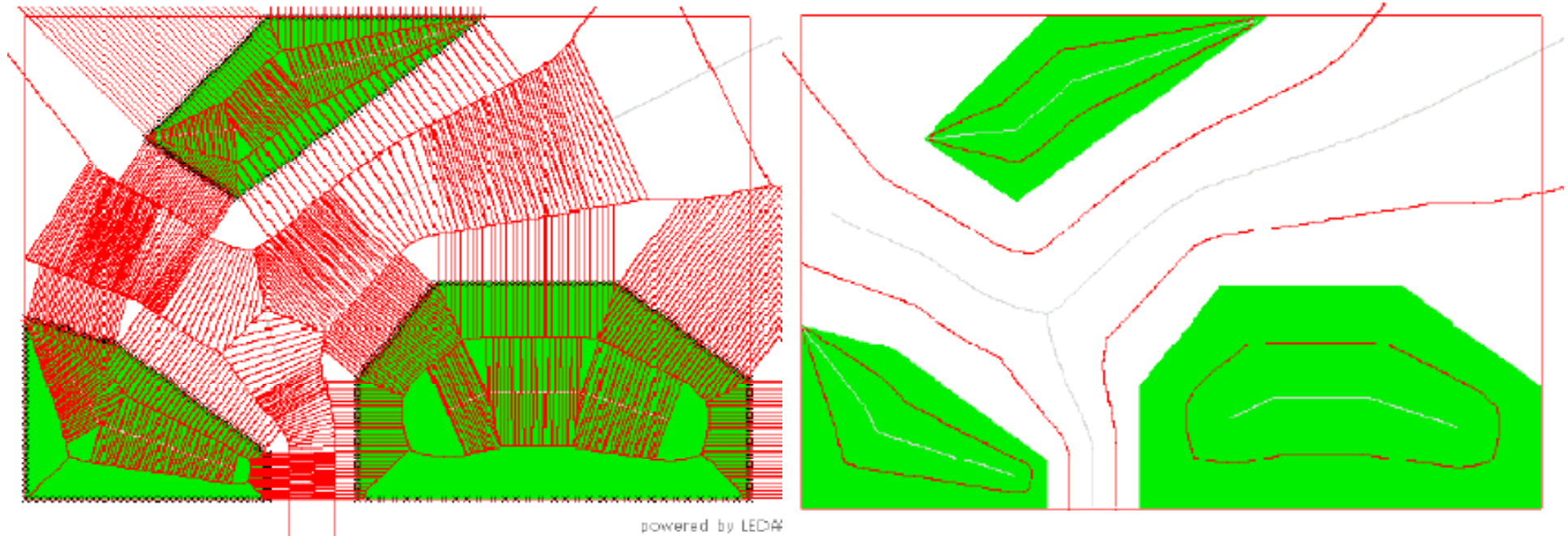
Zimmermann, EWC'2005

powered by LEDA

- A second Voronoi graph can be calculated based upon the merged structure between initial building structure and the first Voronoi graph.
- Further interpolation points are used to get a finer structure of the Voronoi graph

Obstacle Mobility Model

- Modeling a terrain

- Movement graph

  – Voronoi diagram

    - Edges of the obstacles are the Voronoi points (or location points)

- Rout selection

- Signal propagation model

  – Free space

Mobility Model for Mobility Management

OUTLINE

- General aspects: scenarios, cells, ….
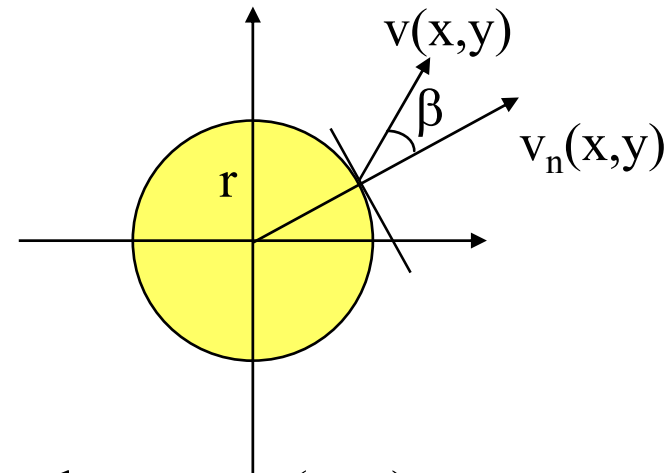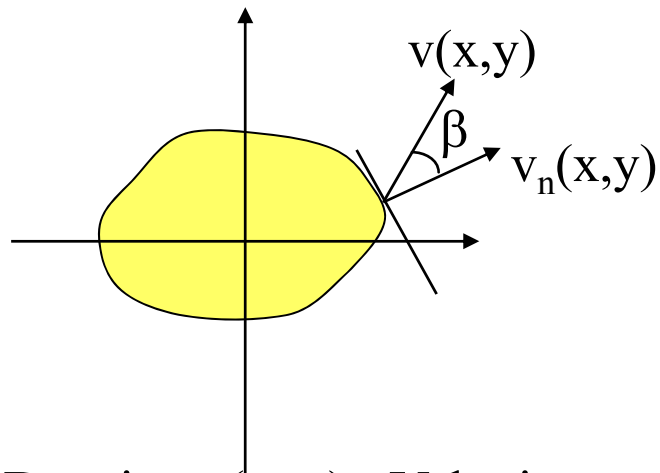- Residence time and call holding times.
    - *Cell and handoff areas.*
- MM in location management.
    - *Random walk.*
    - *Brownian motion.*
    - *Gauss-Markov.*
- **Advanced synthetic mobility models**
- **Macroscopic vision. Transportation theory.**
    - *Fluid flow models.*
    - *Gravity models.*
    - *Rush hour versus busy hour.*
    - *Flow, Speed and Density.*

Fluid flow models

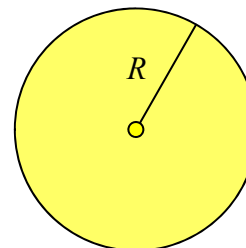- Density $\sigma(x, y)$; Velocity normal to the edge $v_n = v_n(x, y)$ (uniformly distributed in $(0,2\pi)$
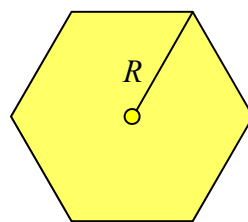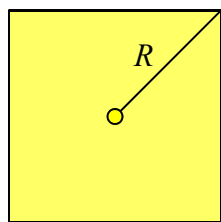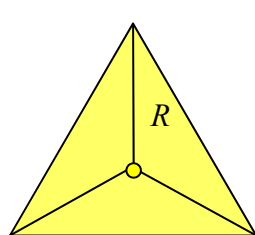
$$v_n(x, y) = \int_{-\pi/2}^{\pi/2} v(x, y)\cos\beta \, d\beta \, / \, 2\pi = v(x, y) / 2\pi$$

$$h = \oint \sigma(x, y)v_n(x, y)dl = \oint \sigma(x, y)v(x, y)dl/2\pi$$

- If density and velocity are constant in the perimeter: $h = \dfrac{\sigma \, v \, L}{\pi}$

Fluid flow versus Markov models

Perimeter: L
Area: A
Density: σ

- Individual behavior: We divide by the total number of users in the Area A

$$\mathrm{h} = \frac{\sigma E[V]L}{\pi} \qquad \rightarrow \qquad \mathrm{cr} = \frac{\sigma E[V]L}{\pi \sigma A} = \frac{E[V]L}{\pi A}$$

Triangle $\qquad \dfrac{L_{tr}}{A_{tr}} = \dfrac{2}{R}; \qquad\qquad \mathrm{cr}_{tr} = 2 \quad \dfrac{2E[V]}{\pi R}$

Square $\qquad \dfrac{L_{sq}}{A_{sq}} = \dfrac{2\sqrt{2}}{R}; \qquad\quad \mathrm{cr}_{sq} = \sqrt{2} \quad \dfrac{2E[V]}{\pi R}$

Hexagon $\quad \dfrac{L_{hx}}{A_{hx}} = \dfrac{4}{\sqrt{3}R}; \qquad\quad \mathrm{cr}_{hx} = \dfrac{2}{\sqrt{3}} \quad \dfrac{2E[V]}{\pi R}$

Circle $\qquad \dfrac{L_{cr}}{A_{cr}} = \dfrac{2}{R}; \qquad\qquad \mathrm{cr}_{cr} = \quad \dfrac{2E[V]}{\pi R}$

Macroscopic level: Trajectories

- During a path from the home to the working place, a mobile will visit some geographical areas covered by several types of cells

Working place

Home
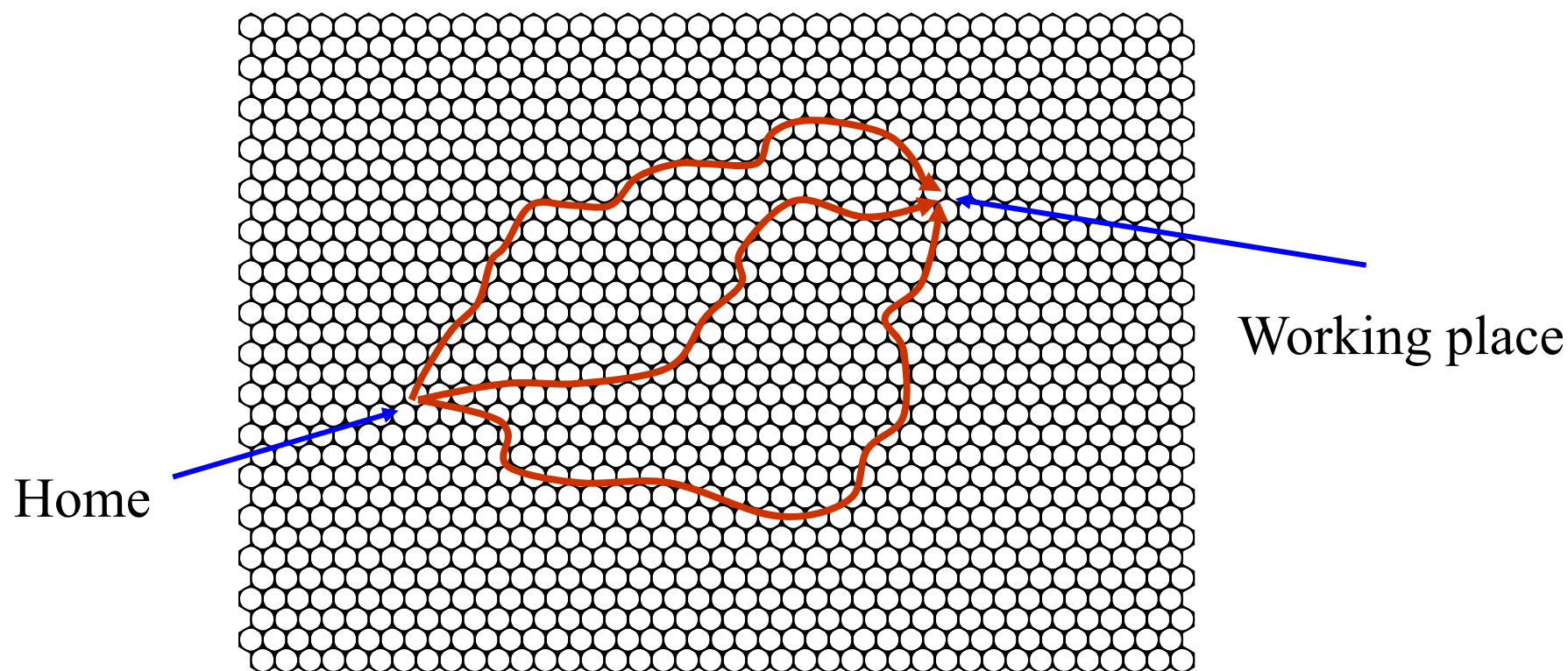
Trajectories in a Manhatan like city

Working place

Home
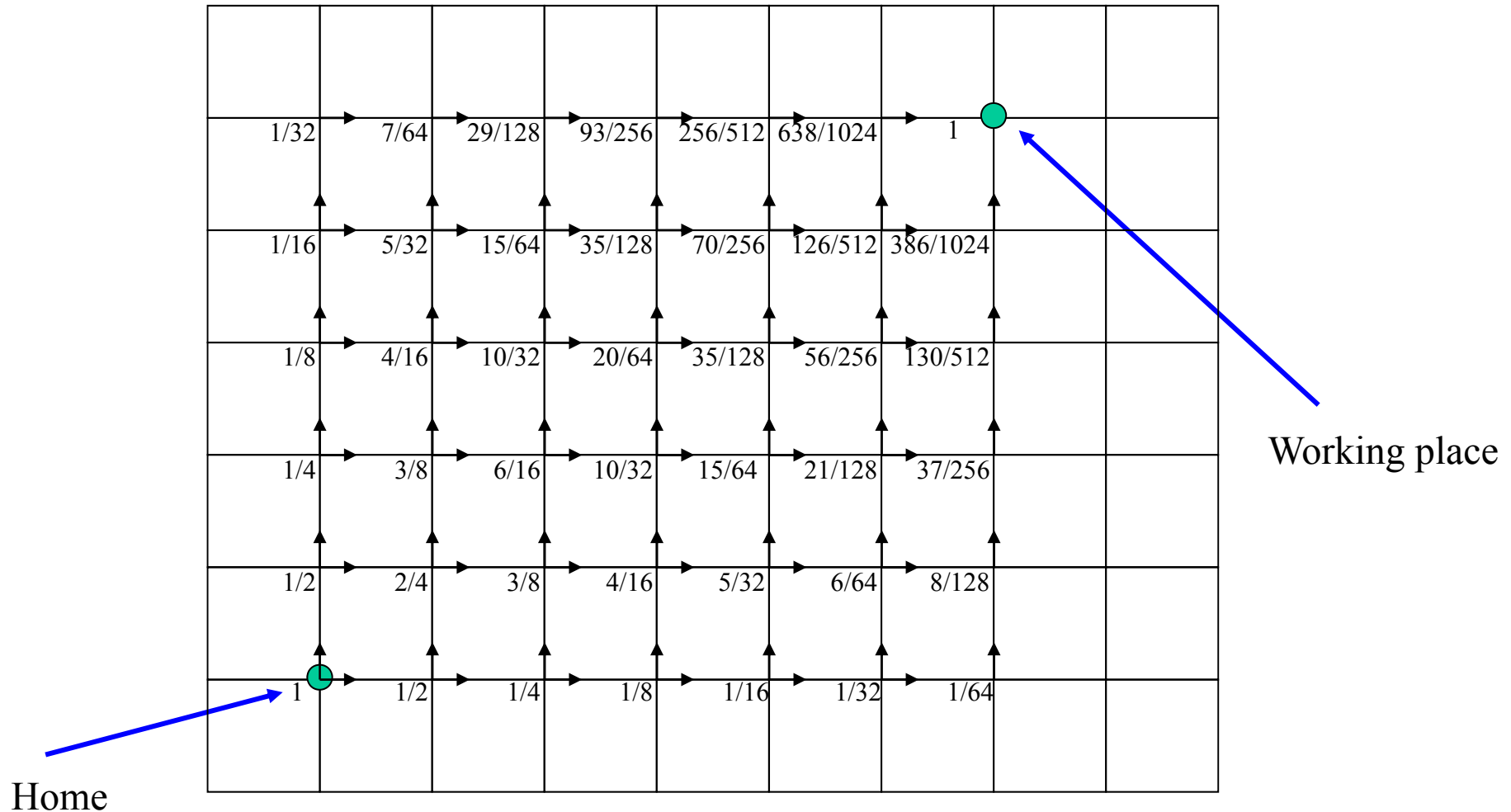
Probability to go North → p=0,5, to go East → q=0,5

Gravity Models

- Transportation theory and fluid flow models can be useful when modeling location management schemes (Markoulidakis -1997, Bejerano, -2000;)

- Attraction points i , j population P_i, P_j
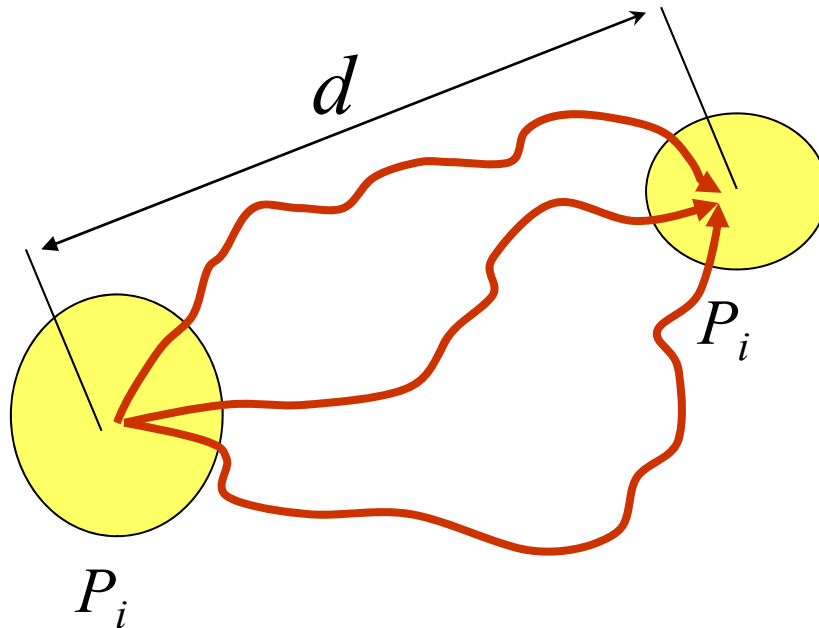
$$T_{i,j} = K_{i,j} P_i P_j$$

- Example: Newton's gravitation law

$$K_{i,j} = C \frac{1}{d^2}$$

Rush hour - Busy hour

—— Share of total traffic (vehicles, pedestrians, ..)

- - - - Teletraffic volume (telephone calls mainly, ..)

RUSH periods (hour)

BUSY periods (hour)

Clock time (24 hours)
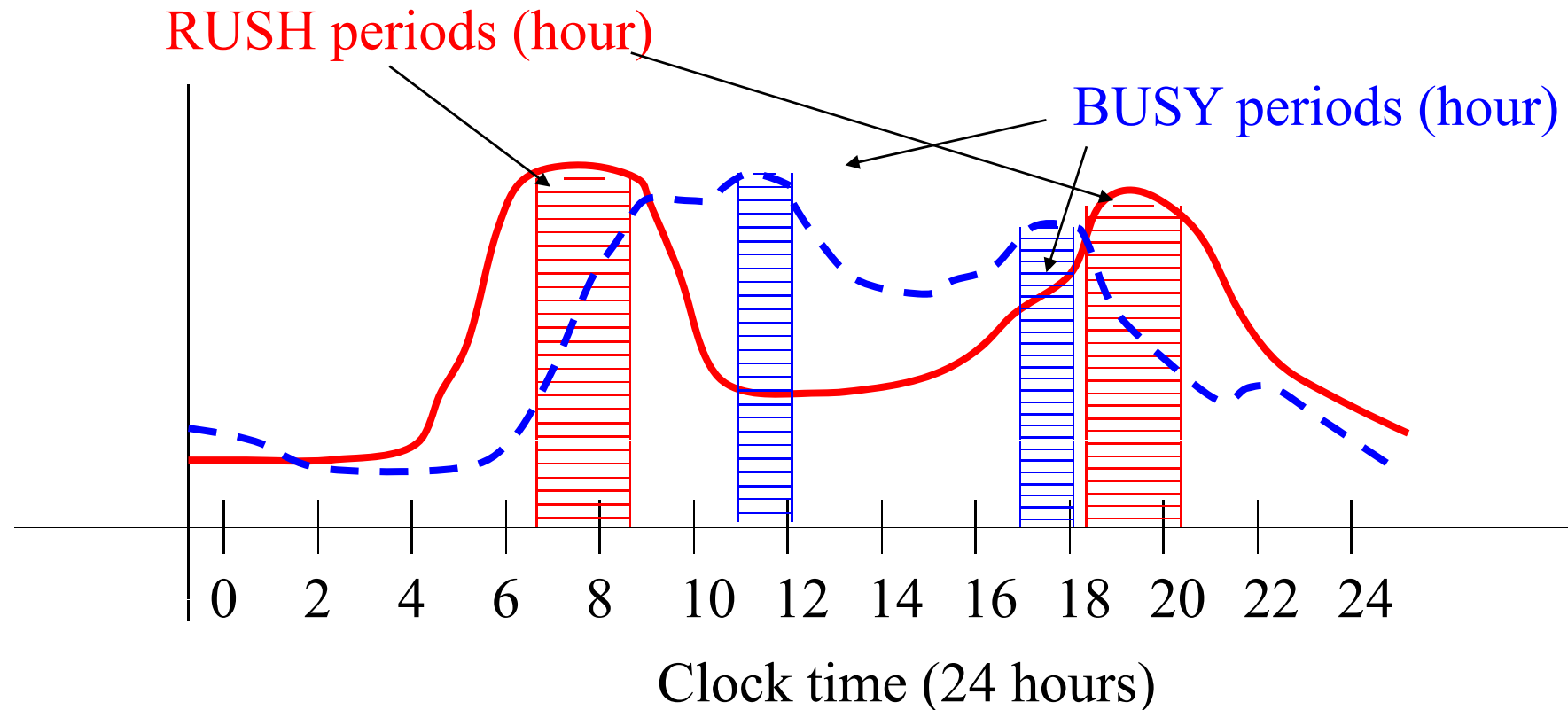
There is no coincidence between RUSH HOURS and BUSY HOURS

Transportation theory –i-

- Framework for fundamental characteristics of traffic flow (A. May, 1990)

Traffic Characteristics	Microscopic (individual units)	Macroscopic (global behavior)
Flow	Time headways	Flow rates
Speed	Individual speeds	Average speed
Density	Distance headways	Density rates

Flows (macroscopic) –i –

- Flow rates (or traffic volume) is the important macroscopic flow characteristic:

  – Defined as the number of vehicles passing a point in a given period of time (morning and evening commuter: **Rush Hours**)

  – The single hour of the day with highest hourly volume: **Peak Hour**

  – Four daily volume parameters are daily used:

    – Average annual daily traffic (AADT)

    – Average annual weekday traffic (AAWT)

    – Average daily traffic (ADT)

    – Average weekday traffic (AWT)

Speed (macroscopic) –i –

- Speed is the second macroscopic parameter describing the state of a traffic stream:
  - It is defined as the rate of motion in distance per unit.
  - Travel time is the time taken to traverse a defined section of roadway.

$$v = \frac{d}{t}$$

- v = speed (Km /hour)
- d = distance traversed.
- t = time to traverse distance d

Traffic density (macroscopic) –i –

- ## Traffic density
  - Defined as the number of vehicles occupying a length (typically 1 mile or 1 Km) of roadway or lane.
    - Minimum: 0 vehicles /Km
    - Maximum: (jam density): 100 -150 vehicles /Km

- ## Traffic density is very difficult to measure

Flow = Speed . Density ?

$$flow = speed \ . \ density ?$$

- Flow, f (the number of vehicles passing a point in a given period of time the distance traveled per time unit).
- Speed, v (the distance traveled per time unit).
- Traffic density, D (the number of vehicles occupying a length of roadway)
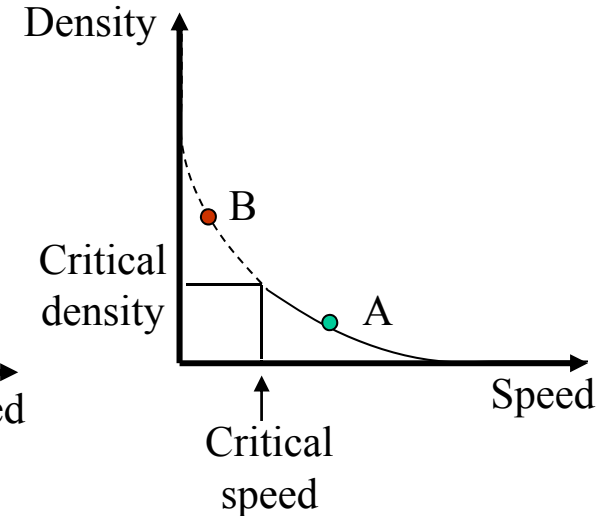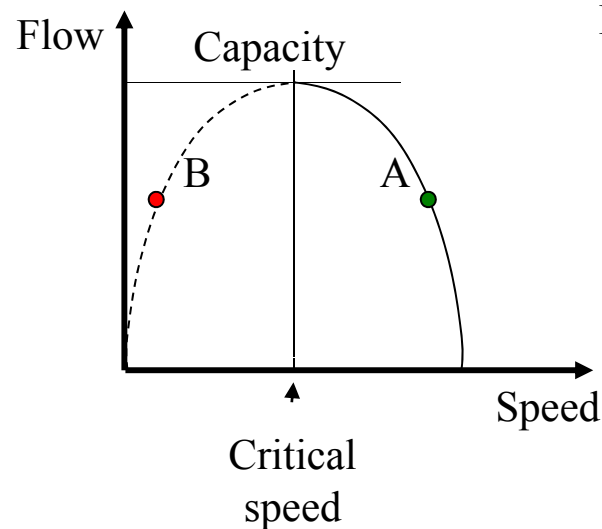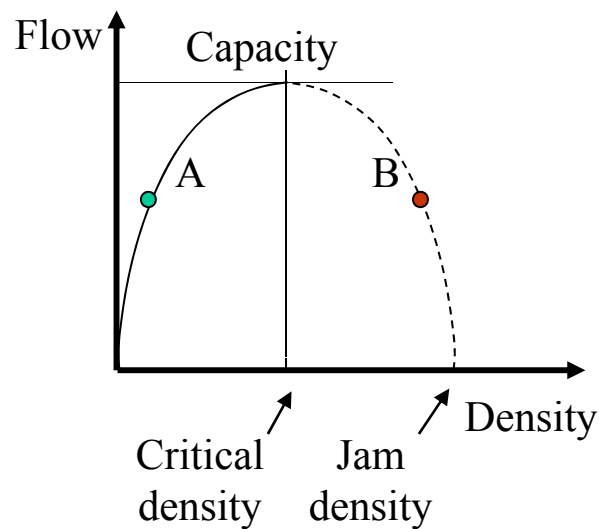
Stable flow

------ Forced flow

Flow-Speed-Density for pedestrians

- Models for pedestrians are similar to those considered for cars. Although, there are some relevant differences:
  - The values and units for speed, flow and density are different
  - The gaps between persons respect their speed are different from cars
  - There is not an integer number of lanes (so clear defined as in roads)
  - Pedestrians stream can be compressed in the transverse dimension

$$flow = speed \, . \, density \, ?$$

Time headways (microscopic) -i-

- Time headways
  - Defined as the elapsed time between the arrival of pairs of vehicles (pedestrian, rail, water, air transportation, ..)
  - Very important for safety conditions, level of service, driver behavior.
  - The average time headway in a traffic lane can be directly related to the density of the lane

$$f = \frac{K}{h} \qquad\qquad f_{hour} = \frac{3600}{h}$$

- f = flow rate (vehicles per hour: f_{hour})
- h = average time headway (seconds per vehicle)

Time headways (microscopic) –ii-

- Three headways distributions are considered for three flow level conditions

Very **low flow level conditions**: Exponential distribution:

$$f_{THW}(t) = \mu e^{-\mu t}$$

High flow level conditions: Constant distribution:

$$f_{THW}(t) = \delta(t - \frac{1}{\mu})$$

Intermediate flow level conditions: Distribution with fitting parameters

Time headways (microscopic) –iii-

- Intermediate flow level conditions: distribution with fitting parameters:
  - Pearson type III distribution
  - Gamma
  - Erlang
  - Sifted negative exponential
  - ….

Pearson type III distribution

$$f_{PR-III}(t) = \frac{\lambda}{\Gamma(k)}[\lambda(t-\alpha)]^{k-1}e^{-\lambda(t-\alpha)}$$

Time headways (microscopic) -iv-

- ## Characterization of platoons (Alpha, Neuts -1995):
  - Based on those, a Markovian Arrival Process (MAP) is built:

  MAP process: C stochastic matrix $\rightarrow$ $C = C_0 + C_1$

  - Transitions of C_0 do NOT involve the arrival of a vehicle.

  - Transitions of C_1 involve the arrival of a vehicle

Time headways (microscopic) -v-

- Characterization of platoons (Alpha, Neuts -1995):
  - **Inter-platoon** headway characterized by a discrete Phase type (PH) distributions. $[\alpha(1), T(1)]$
  - **Intra-platoon** headway characterized by a discrete Phase type (PH) distribution. $[\alpha(2), T(2)]$

$$C_0 = \begin{bmatrix} T(2) & 0 \\ 0 & I \otimes T(1) \end{bmatrix}; \qquad C_1 = \begin{bmatrix} \delta_0 T^0(2)\alpha(2) & \delta \otimes T^0(2)\alpha(1) \\ D^0 \otimes T^0(1)\alpha(1) & D \otimes T^0(1)\alpha(1) \end{bmatrix}$$

$\otimes$:   Kronecker product $\delta_0 + \delta \mathbf{e} = 1 ; \ D^0 + D\mathbf{e} = \mathbf{e}$

δ_0 Is the probability of a platoon consisting of a single vehicle

$T(2)$ Phase transition during an inter-platoon arrival

$I \otimes T(1)$ Phase transition during an inter arrival time within a platoon

Spacing headways (microscopic) -i-

- Spacing headways, d_d
  - Defined as the distance between successive vehicles in a traffic lane elapsed. Also can be applied to pedestrian, rail, water, air transportation, ..
  - Spacing or distance headways: 7- 10 m/vehicles.
  - The average spacing in a traffic lane can be directly related to the density of the lane
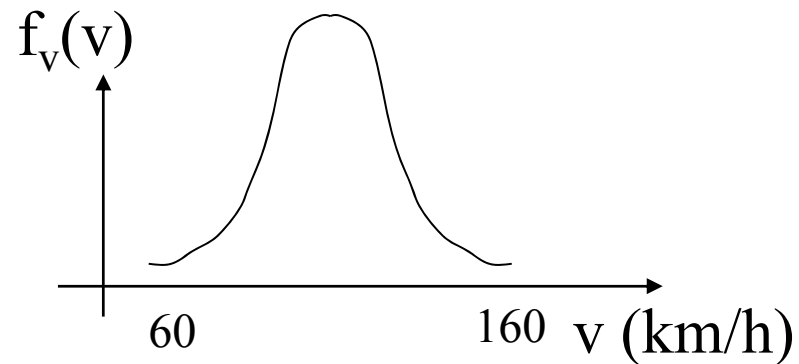
$$d_d = \frac{K}{D}$$

- d_d = average spacing or distance headways
- D = density, vehicles/Km/lane

Speed (microscopic) –i –

- ## Speed in high ways, main roads
  - ### Uniform traffic highway
    - Constant car density
    - Constant car flow
    - Car speed: truncated Gaussian distribution

Some conclusions on MM for MM

- Main Conclusions:

1. Microscopic vision mainly for Call and Handover Management.
   - Good characterization of area (cell, handover) residence time.
   - Good characterization of microscopic traffic transportation (handover batch arrival)

2. Macroscopic vision mainly for Location Management.
   - Macroscopic vision of traffic transportation (Gravity models)

Mobility Models for Mobility Management

- Many thanks indeed for your attention
- Comments or questions?

THE END